

NOTES: The Sine Law

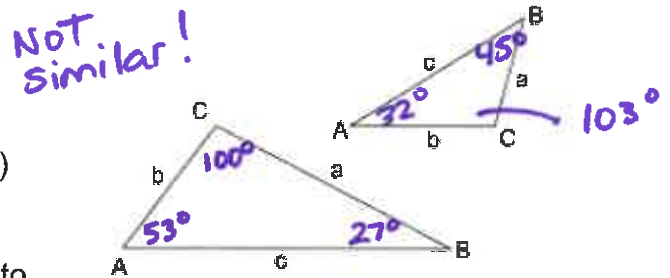
Date: Dec. 8

Neither of the triangles shown are **right** triangles.

We cannot use the Pythagorean theorem or trigonometric ratios (sine, cosine, and tangent) to solve these types of triangles.

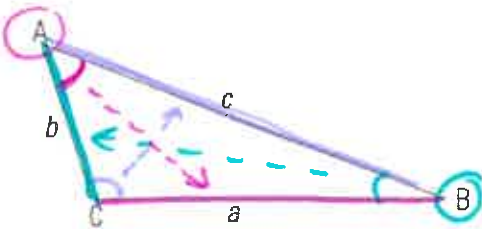
In certain situations, we can use the **sine law** to solve for an unknown length or an unknown angle of an oblique triangle.

↳ does not have a 90° angle (not a RIGHT Δ)



The Sine Law

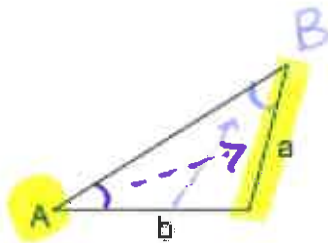
For **all** triangles, the ratio of the sine of an angle to the length of the side opposite is equal for all angle/opposite side pairs.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

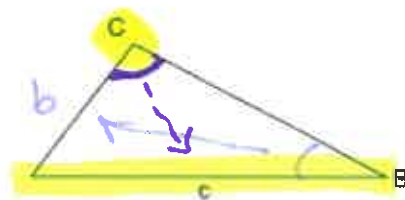
You can use the sine law when you know an **angle/opposite side pair** and one other side or angle.

- If you know two sides and the angle opposite one of them, you can find the angle opposite the other known side.



know $\angle A$ and side a
can calculate $\angle B$
(know side b)

- If you know two angles and the side opposite one of them, you can find the side opposite the other known angle.



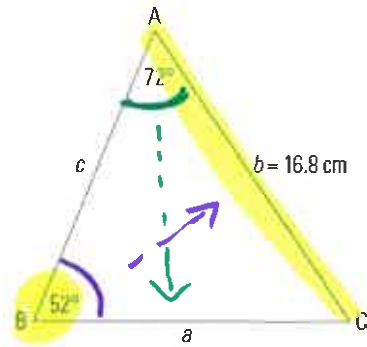
know $\angle C$ and side c
can calculate side b
(know $\angle B$)

Example

Use the sine law to calculate the length of side a.

$$\frac{\sin 52^\circ}{16.8} = \frac{\sin 72^\circ}{a}$$

$$\begin{aligned} a &= \frac{16.8 \times \sin 72^\circ}{\sin 52^\circ} \\ &= 20.276\dots \\ &= \boxed{20.3 \text{ cm}} \end{aligned}$$



Example

Calculate the sizes of $\angle A$ and $\angle C$.

$$\frac{\sin 81^\circ}{123.2} = \frac{\sin A}{91.6}$$

$$\begin{aligned} \sin A &= \frac{\sin 81^\circ \times 91.6}{123.2} \\ &= 0.7344 \end{aligned}$$

$$\begin{aligned} \angle A &= \sin^{-1}(0.7344) \\ &= 47^\circ \end{aligned}$$

$$\boxed{\angle A = 47^\circ}$$

$$\begin{aligned} \text{(OR)} \quad \angle A &= \sin^{-1} \left(\frac{\sin 81^\circ \times 91.6}{123.2} \right) \\ &= 47^\circ \end{aligned}$$

$$\begin{aligned} \angle C &= 180 - 81 - 47 \\ \angle C &= 52^\circ \end{aligned}$$

