Apprenticeship Math 12

NOTES: Simple and Compound Interest

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Date: Jan. 10, 2022

Interest is the cost of borrowing money.

- You earn interest when you lend money (investing or depositing money in a financial institution is like lending money).
- You pay interest when you borrow money.

There are two types of interest: **simple interest** and **compound interest**.

Simple Interest

Simple interest is paid only on the initial amount deposited or borrowed.

The amount of simple interest accumulated on an investment or loan is calculated using the following formula:

$$I = Prt$$

as a decimal ↑ ↑ in years

where: I is the amount of interest earned or due.

P is the principal (the original amount of money invested or borrowed).

r is the annual interest rate, expressed as a decimal.

t is the term of the investment or loan, in years. This is the length of time over which the money is invested or borrowed.

You can calculate the total value of the loan or investment at the end of the term using this formula:

$$A = P + I$$

where: A is the final value of the loan or investment.

$I = Prt$

$A = P + I$

"per annum" means per year

Example

You would like to invest $5000.00 (P) in an account that offers simple interest. Calculate how much the investment would be worth at each of the following rates and terms:

a) 3.00% (r) per annum over a 2-year (t) term;

$3 \div 100 = 0.03$

$I = 5000(0.03)(2)$
$= \$300$

$A = 5000 + 300$
$= \boxed{\$5300}$

b) 1.75% (r) per annum over a 15-month (t) term.

$1.75 \div 100 = 0.0175$ $\quad t = 15 \div 12 = 1.25$ years

$I = 5000(0.0175)(1.25)$
$= \$109.38$

$A = 5000 + 109.38$
$= \boxed{\$5109.38}$

c) 4.75% (r) per annum over a term of 500 (t) days.

$4.75 \div 100 = 0.0475$ $\quad t = 500 \div 365 = 1.3698...$

$I = 5000(0.0475)\left(\frac{500}{365}\right)$
$= \$325.34$

$A = 5000 + 325.34$
$= \boxed{\$5325.34}$

Example

If Bob borrows $3000 (P) at from his uncle and agrees to pay back a total of $3095.50 at the end of 3 years (t), determine the simple interest rate of the loan.

$I = \$95.50$ $\quad (I = A - P = 3095.50 - 3000)$

$I = Prt$

$95.50 = 3000(r)(3)$

$\frac{95.50}{9000} = \frac{9000r}{9000}$

$0.01061 = r$ ← interest rate as a decimal

$\times 100 = \boxed{1.06\%}$

Compound Interest

– interest accumulates faster!

Compound interest is a type of interest that is calculated on the principal plus any interest previously earned. Because you earn (or pay) interest on interest, this results in more interest being earned (or paid) than with simple interest.

Compound interest is much more common than simple interest in the real world.

The Compounding period is the length of time between calculations of interest:

- interest calculated **annually** has 1 compounding period per year
- interest calculated **semi-annually** has 2 compounding period per year
- interest calculated **quarterly** has 4 compounding period per year
- interest calculated **monthly** has 12 compounding period per year
- interest calculated **weekly** has 52 compounding period per year
- interest calculated **daily** has 365 compounding period per year

Compound interest is calculated using the following formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where: ***A*** is the final value of the loan or investment (principal plus interest).
P is the principal.
r is the annual interest rate, expressed as a decimal.
n is the number of compounding periods per year.
t is the term of the investment or loan, in years.

You can calculate the amount of interest earned or due using this formula:

$$I = A - P$$

where: ***I*** is the amount of interest earned or due.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Example

$r = 2.8 \div 100 = 0.028$

Calculate the final value of a deposit of $1000.00 ($P$) invested at a rate of 2.80% per annum for 4 years (t), with the following compounding periods:

a) semi-annual

$n = 2$

$$A = 1000\left(1 + \frac{0.028}{2}\right)^{(2)(4)}$$
$$= 1000(1 + 0.014)^{8}$$
$$= 1000(1.014)^{8}$$
$$= 1000(1.1176\ldots)$$
$$= \$1117.64$$

b) quarterly

$n = 4$

$$A = 1000\left(1 + \frac{0.028}{4}\right)^{(4)(4)}$$
$$= 1000(1.007)^{16}$$
$$= \$1118.08$$

interest grows faster as compounding period gets shorter

c) monthly

$n = 12$

$$A = 1000\left(1 + \frac{0.028}{12}\right)^{(12)(4)}$$
$$= 1000(1.00\overline{23})^{48}$$
$$= \$1118.37$$

It is important to read questions carefully to determine whether you are working with simple or compound interest!

Example

Calculate the interest you would have to pay on $5000.00 loan at 4.0% per annum, compounded annually, if paid it back after 2 years.

P $r = \frac{4}{100} = 0.04$

$n = 1$ $t = 2$

$$A = 5000\left(1 + \frac{0.04}{1}\right)^{(1)(2)}$$

$$= 5000(1.04)^2$$

$$= \$5408$$ ← pay back

$$I = 5408 - 5000$$

$$= \$408$$ ← cost to borrow

The Rule of 72

The Rule of 72 is an easy way to estimate how long it will take you to double your investment if it is compounded annually.

$$Years\ to\ double\ investment = 72 \div interest\ rate\ as\ a\ percent$$

Example

Approximately how long will it take an investment of $5000.00, invested at a rate of 3.75% per annum, compounded annually, to double in value?

* 3.75%

$$\text{time to double} = \frac{72}{3.75}$$

$$= 19.2 \Rightarrow$$ it will take just over 19 years to double