

Exploring Pascal's Triangle

Name: _____

Date: _____

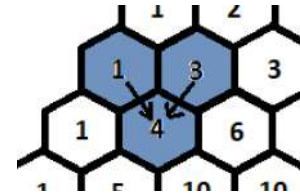
Pascal's Triangle is named after *Blaise Pascal*, a famous French Mathematician and Philosopher) from the 1600's.

The triangle was discovered and used much earlier in India, Iran, China, Germany, Greece and Italy.

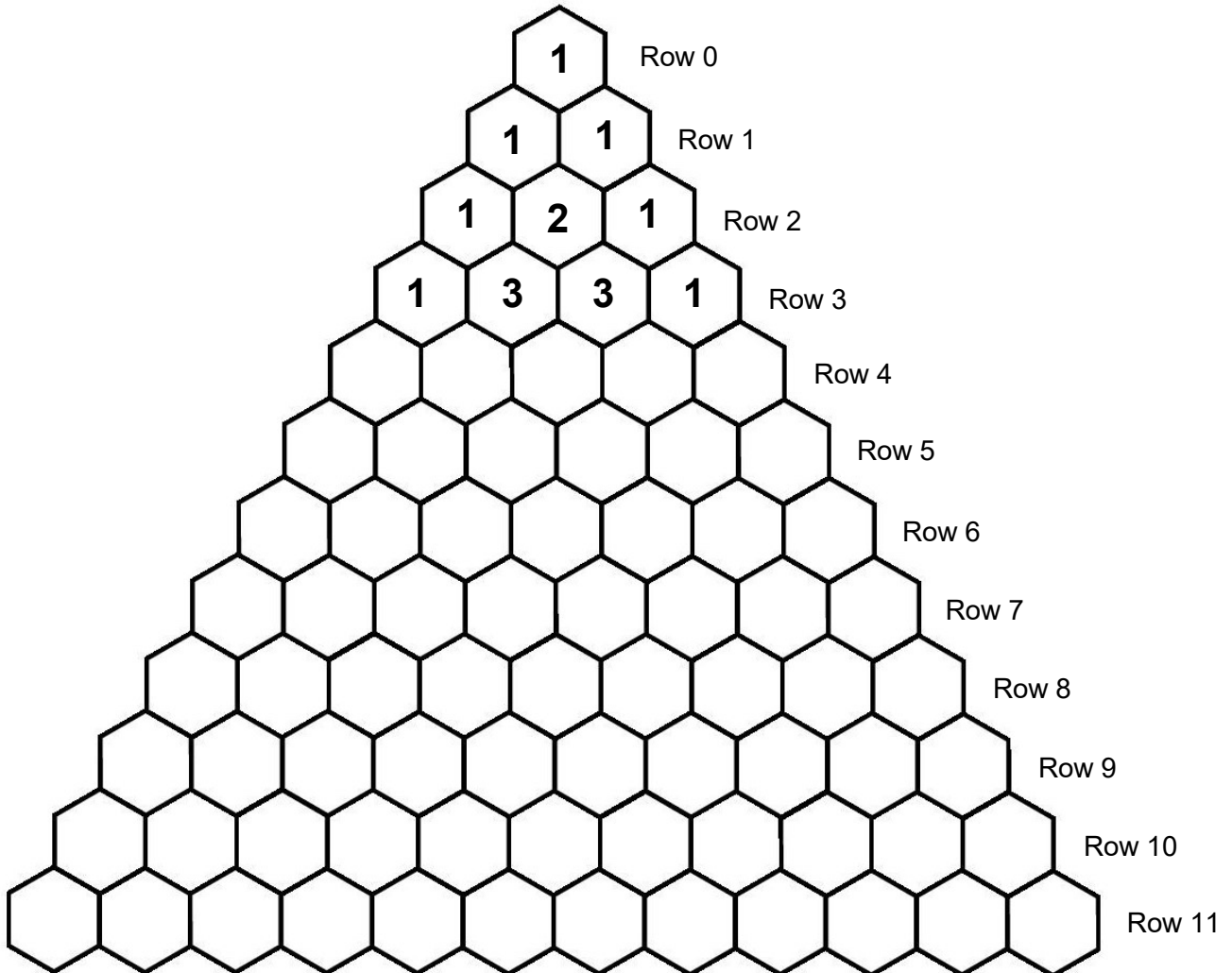
To build the triangle, start with "1" at the top, then continue placing numbers below it in a triangular pattern.

Each number is the sum of the numbers directly above it.

For example, $1+3 = 4$, in the image to the right.



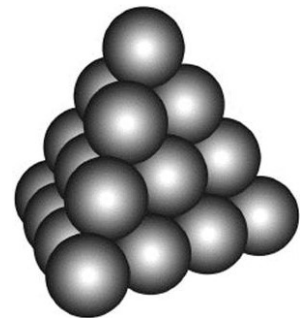
1. Fill in the missing numbers in Pascal's Triangle below.



2. Find the sum of each row in Pascal's Triangle. Describe the pattern of the sum of each row.

3. There are a number of patterns that show up in the diagonals of Pascal's Triangle. List and find the following:

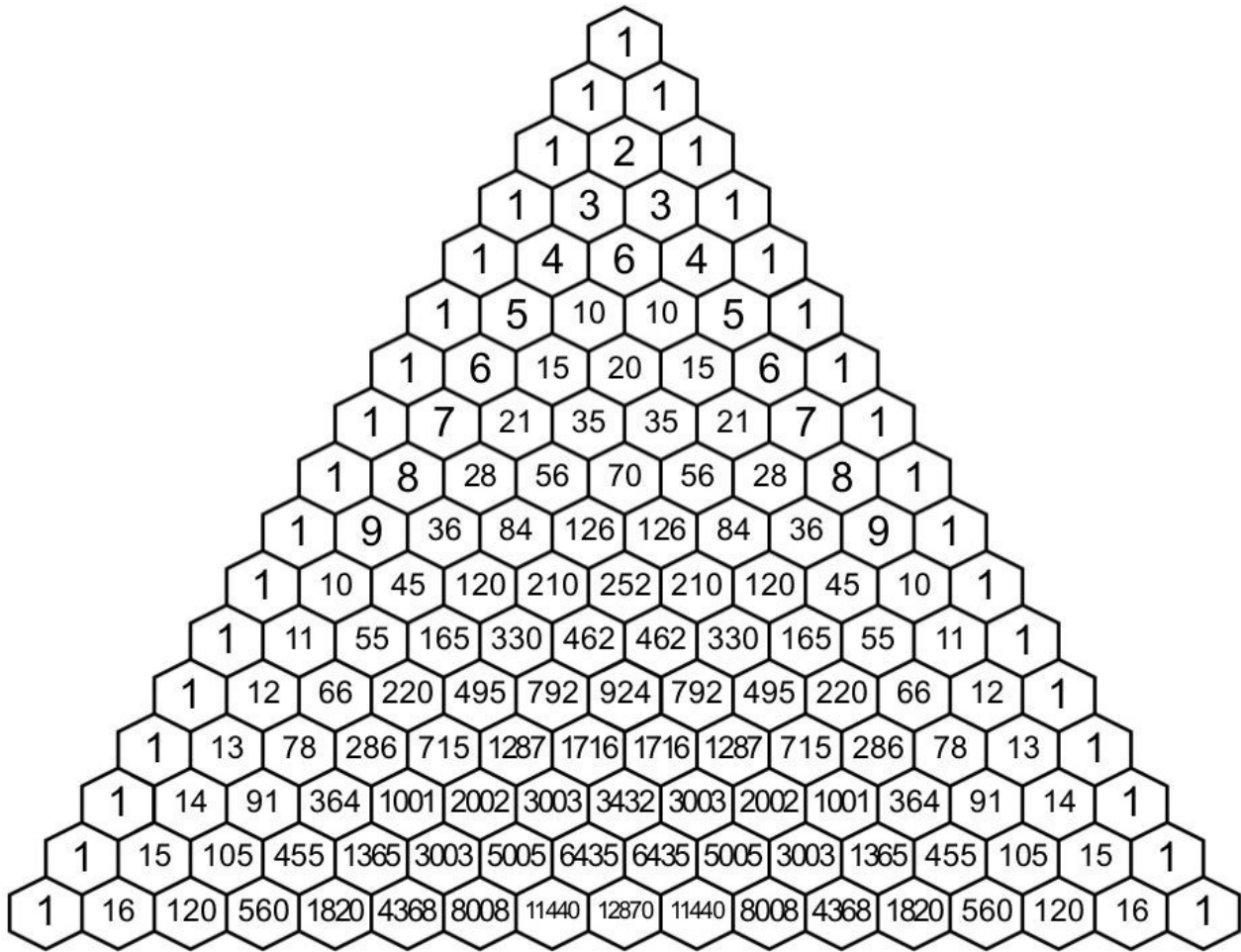
- ones
- natural numbers
- triangular numbers (*can be arranged in an equilateral triangle*)
- tetrahedral numbers (*can be arranged in a tetrahedron – a pyramid with a triangular base and three sides*)



4. Do you notice any patterns related to the number 11?

5. Fibonacci's famous sequence (0, 1, 1, 2, 3, 5, 8, 13...) can be found in Pascal's triangle. Can you find the Fibonacci numbers? (Hint: they require some adding.)

6. Shade in all of the odd numbers in Pascal's triangle below. Is there a pattern?



7. Expand the following binomials.

$$(x + 1)^0 =$$

$$(x + 1)^1 =$$

$$(x + 1)^2 =$$

$$(x + 1)^3 =$$

Can you find the expansion of $(x + 1)^{11}$ using Pascal's Triangle?

8. Pascal's Triangle has applications for probability.
If you toss a coin three times, there is one way to get all heads (HHH), three ways to get two heads and a tail (HHT, HTH, THH), three ways to get one head and two tails (HTT, THT, TTH) and one way to get all tails (TTT).
- Row 3 illustrates the number of possibilities for each scenario (1, 3, 3, 1). The sum of the row is the total number of choices (8). The probability for tossing two heads and one tail is $\frac{3}{8}$.
- Using Pascal's triangle, can you find the probability for throwing two heads and two tails with four coins?

9. Pascal's Triangle can also be used to find combinations.
- In how many ways can you choose teams of five from nine people? To solve this problem using Pascal's Triangle, go down to row 9 and then over five places. The number in that cell is your answer (126).
- Using Pascal's triangle, can you find the number of ways you can choose teams of 6 from 15 people?

Adapted from the Following Sources

- <https://www.mathsisfun.com/pascals-triangle.html>
- <https://www.youcubed.org/wp-content/uploads/2017/07/WIM-Day-4-gr-5-9-vF.pdf>
- Investigating Pascal's Triangle (TpT resource prepared by Nemecek Learning Academy)