7.4 – Characteristics of Logarithmic Functions with Base 10 and Base *e*

Date: Apr. 27

Definitions:

- **Logarithmic function:** A function of the form $f(x) = a\log_b x$ where $b > 0$, $b \neq 1$ and $a \neq 0$, and a and b are real numbers.

A logarithm "undoes" an exponent.

$$log_2(16) = ?$$

The log asks us how many of the base need to be multiplied together to give the value in brackets. The above reads as "How many times do we multiply 2 by itself to get 16?

$2 \times 2 \times 2 \times 2$ base

$2^4 = 16$ base

$\log_{2}(16) = 4$ base

A logarithm can be written with any base; however, the most common bases are base 10 and base e.

$$log_{10}(x) \quad \underline{\text{or}} \quad log_e(x)$$

natural log.

$log_{10}(x) = \log(x)$ $\boxed{\text{log}}$ on calculator

$log_e(x) = \ln(x)$ $\boxed{\text{ln}}$ on calculator

Example: Solve the following logarithms:

$log_5(625) = 4$ $5 \times 5 \times 5 \times 5$ $5^{4} = 625$	$log_2(64) = 6$ $2 \times 2 \times 2 \times 2 \times 2 \times 2$ $2^{6} = 64$
$log_{10}(100) = 2$ can use calculator for base 10!	$log_3(243) = 5$ $3 \times 3 \times 3 \times 3 \times 3$ $3^{5} = 243$

In general, the relationship between a logarithm and an exponential can be translated as such:

$$a^x = y$$

exponent

base

$$log_a(y) = x$$

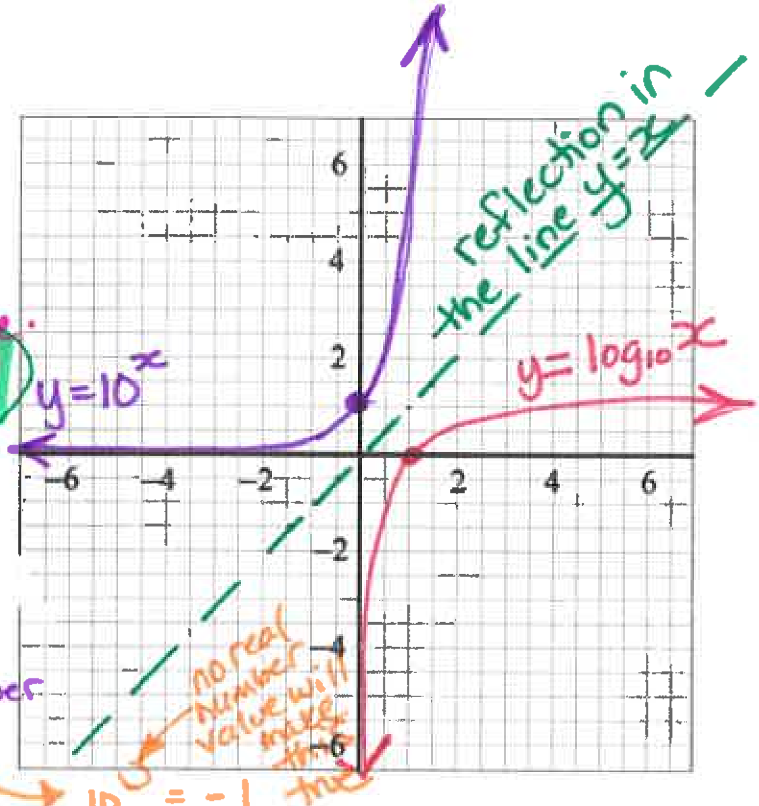
At this level, we focus almost exclusively on how the equation and graph can be used to predict the characteristics of the function.

Graph the following using your graphing calculator:

$y = 10^x$

$y = log_{10}x$

No matter what we put in for x (input) y is never negative (output)



$y=10^x$

x	y
-1	$\frac{1}{10}=0.1$
0	1
1	10
2	100

$y = \log_{10} x$

$10^y = x$

x	y
-1	no real answer
0	error
0.5	-0.301
1	0

$10^y = -1$

Some logarithms use a base of Euler's number, 'e', instead of a base of 10. When we use 'e' as our base, we call this a natural logarithm, and represent it using 'ln' instead of 'log'.

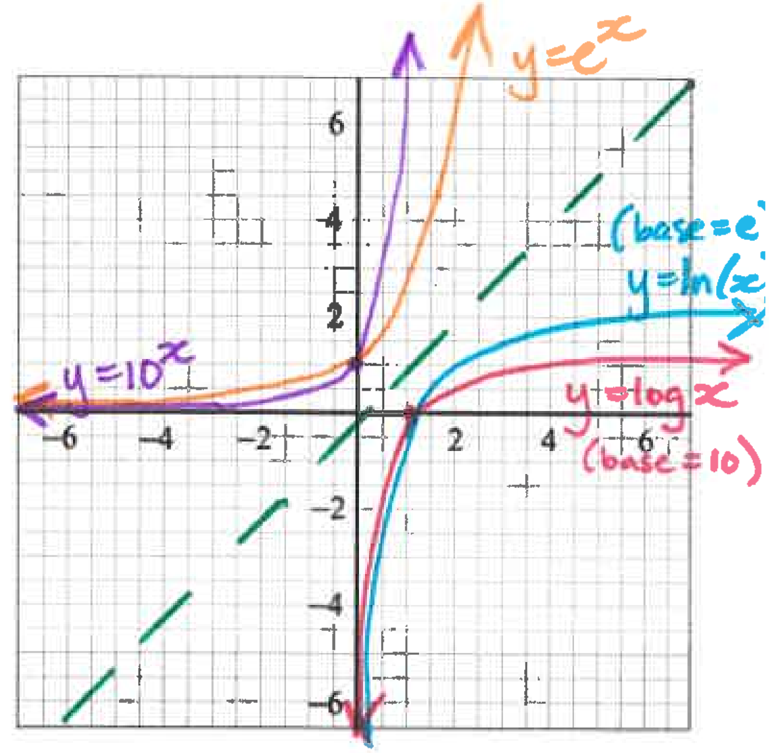
Graph the following using your graphing calculator:

$y = log_{10}x$

$y = \ln(x)$

$y = e^x$

$y=10^x$



Apr.27

What effect does the change in base have on the graph? of a logarithmic function?

For logarithms, higher base → shallower graph.

We can also add a coefficient to the logarithm. We solve the logarithm as before, and we treat the coefficient like a typical coefficient.

Example: Solve the following logarithms:

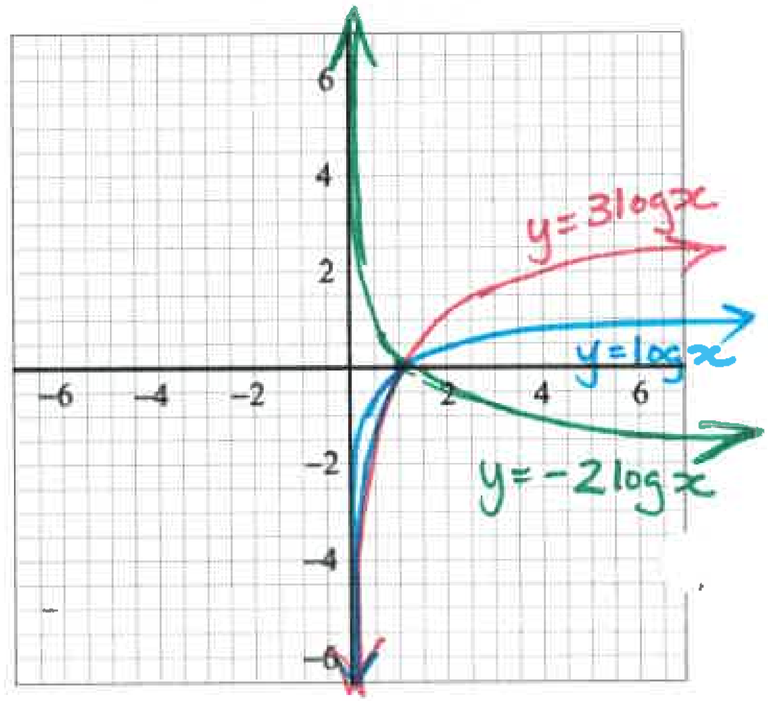
$3log_{10}(1000) = 3(3)$ coeff $= 9$	$-5log(100) = -5(2)$ $= -10$

Graph the following using your graphing calculator:

$$y = log_{10}x$$

$$y = 3log_{10}x$$

$$y = -2logx$$



What effect does the coefficient have on the graph?

- the coefficient amplifies the graph

stretches in the y direction

if coefficient $a>0$, the function increases

if coefficient $a<0$, the function decreases

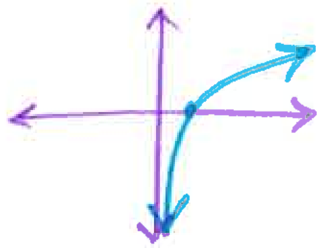
$f(x) = a \log_b x$

Example: Predict the x-intercept, the number of y-intercepts, the end behaviour, the domain and the range of the following function, and state whether the function is increasing or decreasing.

$y = 2log_{10}(x)$

$a = 2$ ← increasing

$b = 10$



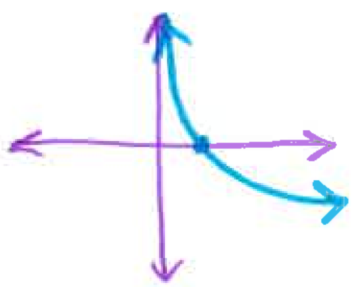
- x-intercept = 1
- no y-intercept.
- QIV to QI (increasing) $a>0$
- $\{x \mid x>0, x \in R\}$
- $\{y \mid y \in R\}$

Example: Predict the x-intercept, the number of y-intercepts, the end behaviour, the domain and the range of the following function, and state whether the function is increasing or decreasing.

$y = -4\ln(x)$

$a = -4$ ← decreasing

$b = e \approx 2.718...$



- x-intercept = 1
- no y-intercept
- QI to QIV (decreasing) $a<0$
- $\{x \mid x>0, x \in R\}$
- $\{y \mid y \in R\}$

$f(x) = a(b)^x$ $\qquad$ $f(x) = a\log_b x$

Example: Match each function with the corresponding graph. Explain your reasoning.

i) $y = -(5)^x$ $\qquad$ ii) $y = 1.5(1.4)^x$ $\qquad$ iii) $y = 5\log(x)$ $\qquad$ iv) $y = -3\ln x$

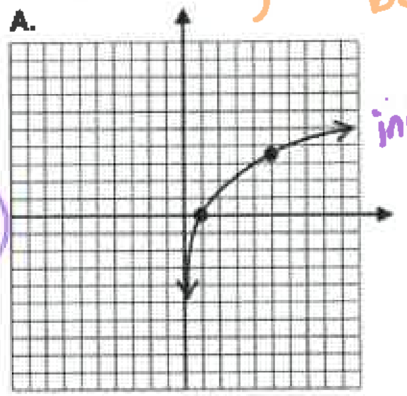
i) exp, $a < 0$, y will always be negative.

ii) exp. $a > 0, b > 1$, increasing, y will always be positive.

iii) log, $a > 0$, increasing

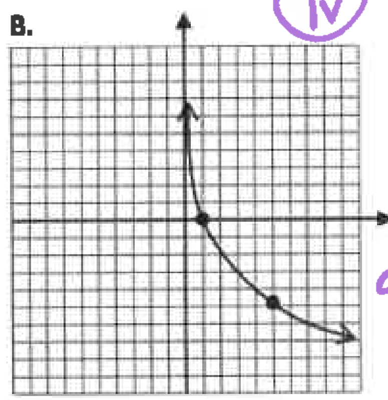
iv) log, $a < 0$, decreasing.

A. iii



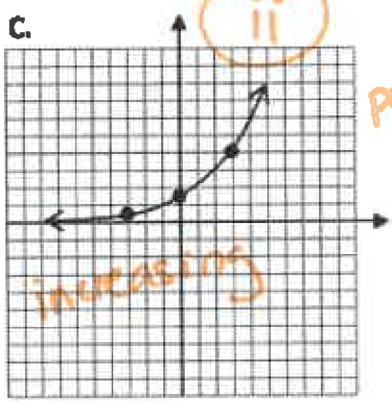
increasing

B. iv



decreasing.

C. ii

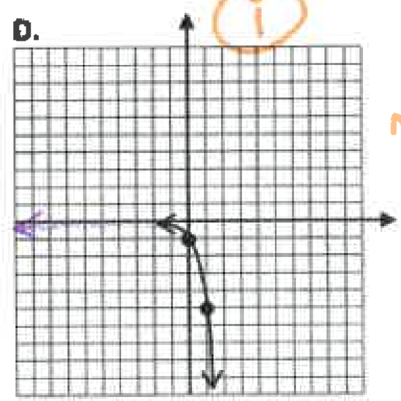


positive y

increasing

exp → doesn't cross x-axis

D. i



negative y

exp. → doesn't cross x-axis.