7.2 – Relating the Characteristics of an Exponential Function to Its Equation

Date: Apr. 25

Definitions:

- e: The symbol e is a constant known as Euler's constant.
  It is an irrational number (like pi) that approximates 2.718.....
  Euler's constant occurs naturally in some situations where a quantity increases continuously, such as increasing population.
- **Asymptote:** a line that continually approaches a given curve but does not meet it at any finite distance. ← gets close but never touches

An exponential function takes the form $y = a(b)^x$, where a ≠ 0 and b > 0, b ≠ 1.

The variable 'b' is known as the base of the exponential; it determines how quickly the exponential increases.

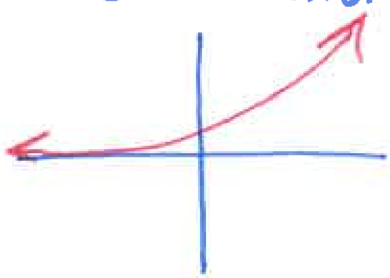
The variable 'a' is the coefficient; determines the y-intercept.

We can use the equation of the exponential to predict the behaviour.

Example: Predict the number of x-intercepts, the y-intercept, the end behaviour, the domain and the range of the following function:

$a = 1$
$b = e \approx 2.718...$

$y = e^x$



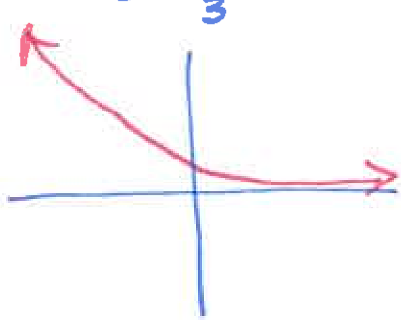
x-axis is an asymptote

- no x-intercept
- y-int = 1
- QII to QI, increasing curve ($b>1$)
- $\{x \mid x \in R\}$
- $\{y \mid y>0, y \in R\}$

Example: Predict the number of x-intercepts, the y-intercept, the end behaviour, the domain, the range, and whether this function is increasing or decreasing:

$a = 9$
$b = \frac{2}{3}$

$y = 9(\frac{2}{3})^x$

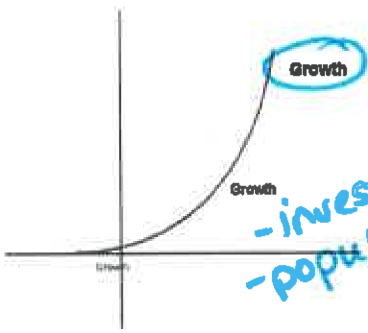


- no x-intercept
- y-int = 9
- QII to QI, decreasing curve ($0<b<1$)
- $\{x \mid x \in R\}$
- $\{y \mid y>0, y \in R\}$

Summarize the relationship between the base and whether a function is increasing or decreasing: if $b>1$, increasing function (steeper as $b \to \infty$)

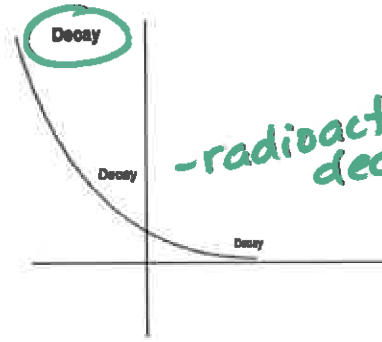
if $0<b<1$, decreasing function (steeper as $b \to 0$)

Exponential growth is when the shape of a graph increases exponentially reading from left to right.



-investments
-population

Exponential decay is when the shape of a graph decreases exponentially reading from left to right.



-radioactive decay

Example: Match the exponential equation with its corresponding graph (without using the graphing calculator).

$b>1$ inc

i. $y = \frac{1}{3}(3)^x$ ← y-int $= \frac{1}{3}$

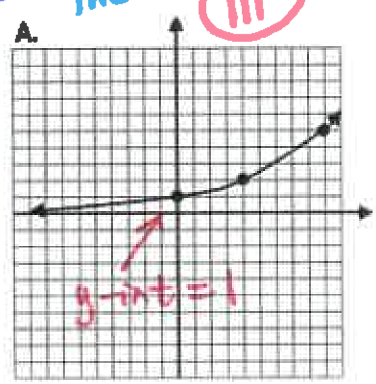
ii. $y = 3(4)^x$ ← y-int $= 3$

iii. $y = 1(1.2)^x$ — y-int $= 1$

iv. $y = 3(\frac{1}{3})^x$

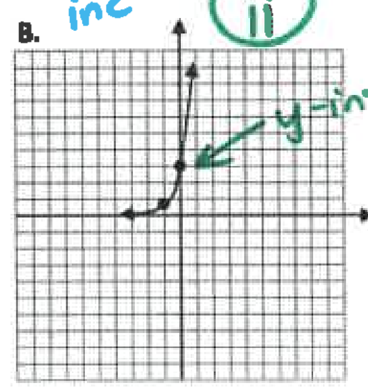
$0<b<1$ decreasing

A. inc iii



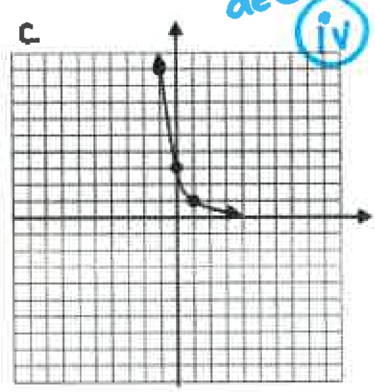
y-int $= 1$

B. inc ii

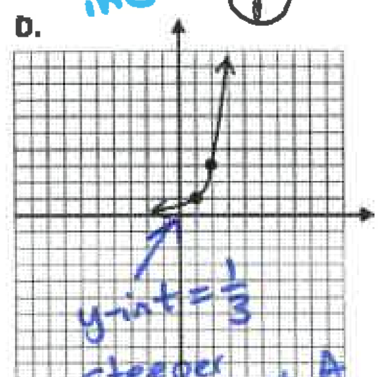


y-int $= 3$

C. dec iv



D. inc i



y-int $= \frac{1}{3}$

steeper than graph A