7.1 – Exploring the Characteristics of Exponential Functions

Date: Apr. 24

Definitions:

- **Exponential function:** A function of the form $f(x) = a(b)^x$ where $a \neq 0$, $b > 0$, and $b \neq 1$.

Explore the Math *(see p. 436)*

Plot a graph for each of the exponential functions represented in the tables below.

$f(x) = 10^x \rightarrow f(x) = 1(10)^x$

$a = 1$
$b = 10$

$(10)^{-3} = \frac{1}{10^3}$

x	$f(x)$
-3	$10^{-3} = 0.001$
-2	$10^{-2} = 0.01$
-1	$10^{-1} = 0.1$
0	$10^{0} = 1$
1	$10^{1} = 10$
2	$10^{2} = 100$
3	$10^{3} = 1000$

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$g(x) = 2(5)^x \rightarrow$ $a = 2$, $b = 5$

x	$f(x)$
-3	$2(5)^{-3} = 0.016$
-2	$2(5)^{2} = 0.08$
-1	$2(5)^{1} = 0.4$
0	$2(5)^{0} = 2$
1	$2(5)^{1} = 10$
2	$2(5)^{2} = 50$
3	$2(5)^{3} = 250$

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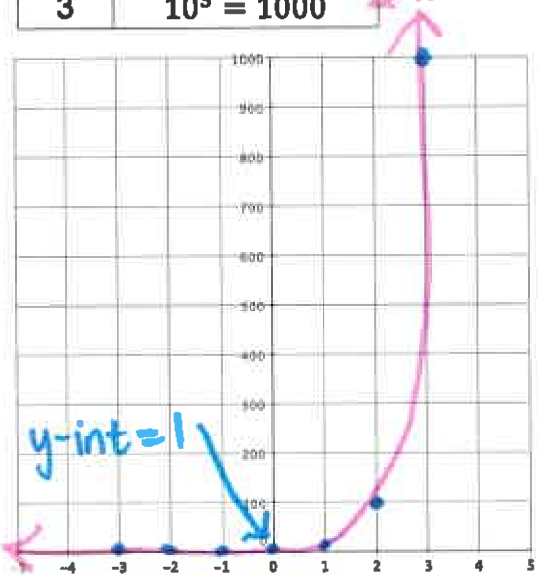
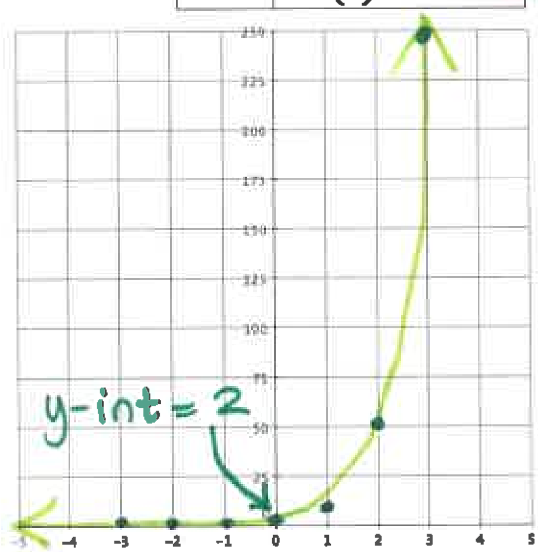




Describe characteristics of these functions:

- The number of x-intercepts → no x-intercept
- The y-intercept → one y-intercept, y-int = a or (0, a)
- The end behaviour → QII to QI (increasing curve)
- The domain → $\{x \mid x \in R\}$
- The range → $\{y \mid y > 0, y \in R\}$

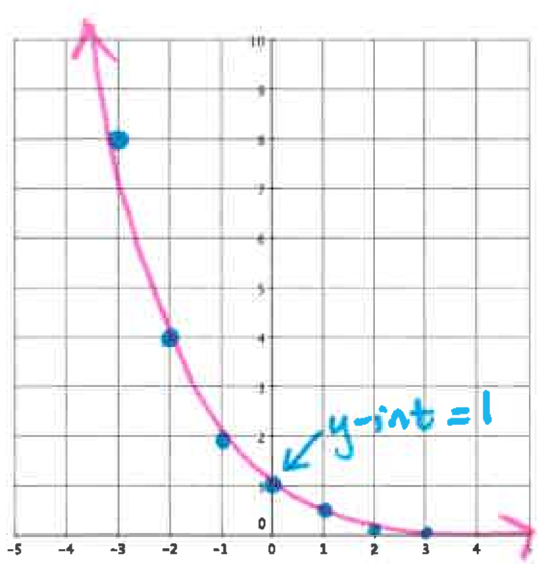
Plot a graph for each of the exponential functions represented in the tables below.

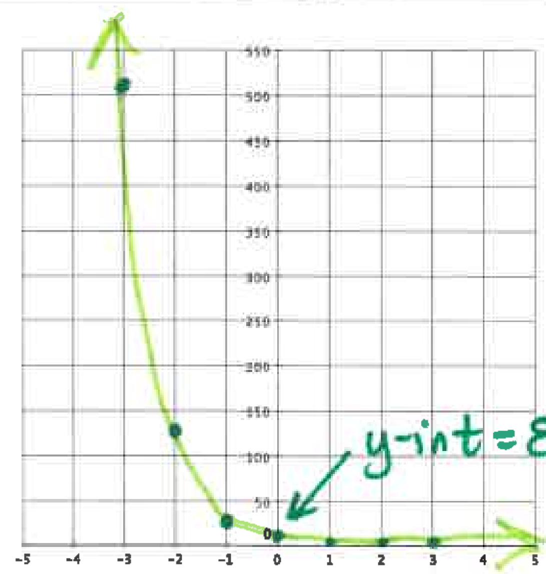
$f(x) = (\frac{1}{2})^x \rightarrow a = 1, \; b = \frac{1}{2}$

x	$f(x)$	
-3	$(\frac{1}{2})^{-3} = (\frac{2}{1})^3 = 8$	$\times\frac{1}{2}$
-2	$(\frac{1}{2})^{-2} = (\frac{2}{1})^2 = 4$	$\times\frac{1}{2}$
-1	$(\frac{1}{2})^{-1} = (\frac{2}{1})^1 = 2$	$\times\frac{1}{2}$
0	$(\frac{1}{2})^0 = 1$	$\times\frac{1}{2}$
1	$(\frac{1}{2})^1 = 0.5$	$\times\frac{1}{2}$
2	$(\frac{1}{2})^2 = 0.25$	$\times\frac{1}{2}$
3	$(\frac{1}{2})^3 = 0.125$	

$g(x) = 8(\frac{1}{4})^x \rightarrow a = 8, \; b = \frac{1}{4}$

x	$f(x)$	
-3	$8(\frac{1}{4})^{-3} = 8(\frac{4}{1})^3 = 8(64) = 512$	$\times\frac{1}{4}$
-2	$8(\frac{1}{4})^{-2} = 8(\frac{4}{1})^2 = 8(16) = 128$	$\times\frac{1}{4}$
-1	$8(\frac{1}{4})^{-1} = 8(\frac{4}{1})^1 = 8(4) = 32$	$\times\frac{1}{4}$
0	$8(\frac{1}{4})^0 = 8(1) = 8$	$\times\frac{1}{4}$
1	$8(\frac{1}{4})^1 = 8(\frac{1}{4}) = 2$	$\times\frac{1}{4}$
2	$8(\frac{1}{4})^2 = 8(\frac{1}{16}) = 0.5$	$\times\frac{1}{4}$
3	$8(\frac{1}{4})^3 = 8(\frac{1}{64}) = 0.125$	





Describe the following characteristics of these functions:

- The number of x-intercepts → no x-intercept
- The y-intercept → one y-intercept, y-int = a or (0, a)
- The end behaviour → QII to QI (decreasing curve)
- The domain → $\{x | x \in R\}$
- The range → $\{y | y > 0, y \in R\}$

In Summary

Key Ideas

- An exponential function has the form $f(x) = a(b)^x$, where x is the exponent and $a \neq 0$, $b > 0$, and $b \neq 1$.
- All exponential functions of the form $f(x) = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, have the following characteristics:

Number of x-intercepts	0
y-intercept	a
End Behaviour	Curve extends from quadrant II to quadrant I.
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y > 0, y \in \mathbb{R}\}$

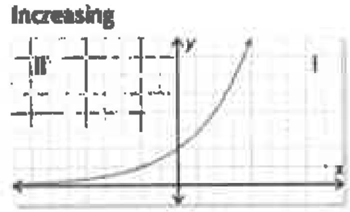
Need to Know

- There are two different shapes of the graphs of an exponential function of the form $f(x) = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$:

  - Case 1: An increasing function; the curve extends from quadrant II to quadrant I.

  - Case 2: A decreasing function; the curve extends from quadrant II to quadrant I.

Increasing



Decreasing