6.2 – Characteristics of the Equations of Polynomial Functions

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Definitions:

- **Standard form:** The standard form for a function is the standard way the function can be written, with the term with the highest exponent first and the terms in order of decreasing exponents.
  - The standard form for a linear function is $f(x) = ax + b$ where $a \neq 0$
  - The standard form for a quadratic function is $f(x) = ax^2 + bx + c$ where $a \neq 0$
  - The standard form for a cubic function is $f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$
- **Leading coefficient:** The coefficient of the term with the highest exponent in a polynomial function in standard form. (a)
- **Constant term:** The term in a polynomial without a variable, which moves the graph vertically.

Example: Determine the following characteristics of each function using its equation:

Number of possible x-intercepts	Domain
y-intercept	Range
End behaviour	Number of possible turning points

a) $f(x) = 2x - 6$ linear → degree = 1 a = 2 constant = −6

- one x-intercept
- y-int = −6 (constant)
- positive leading coeff. (positive slope) ↳ line extends QIII to QI
- $\{x \mid x \in R\}$
- $\{y \mid y \in R\}$
- 0 turning points

b) $f(x) = -3x^2 - x + 6$ quadratic → degree = 2 a = −3 constant = 6

- negative leading coeff. (parabola opens down) and positive constant (y-int above x-axis) ⟹ 2-x intercepts
- y-int = 6
- curve extends QIII to QIV
- $\{x \mid x \in R\}$
- $\{y \mid y \leq$ maximum, $y \in R\}$ (don't know vertex)
- 1 turning point

c) $f(x) = 2x^3 + 5x^2 - 3x - 4$

cubic → degree = 3 $\quad a = 2$ $\quad$ constant = −4

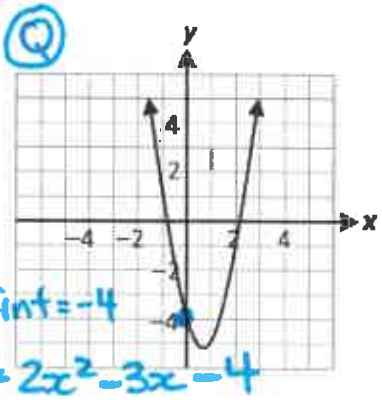
- 1, 2 or 3 x-intercepts (can't tell which from equation)
- y-int = −4
- positive leading coeff.
  - ↳ curve extends from QIII to QI
- $\{x \mid x \in R\}$
- $\{y \mid y \in R\}$
- 0 or 2 turning points (can't tell from equation)

Example: Match each graph with the correct polynomial function. Justify your reasoning.

y-int = −4

C $g(x) = -x^3 + 2x^2 - 6x - 4$	$j(x) = 2x^2 - 3x - 4$ Q	L $p(x) = -1 - x$
L $h(x) = (x+3)(x-1)$ Q	L $k(x) = -4x - 1$ ↑ steeper slope!	C $q(x) = x^3 - 3x^2 - 3$

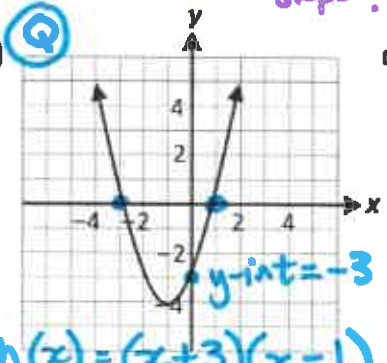
a) Q



y-int = −4

$j(x) = 2x^2 - 3x - 4$

b) Q

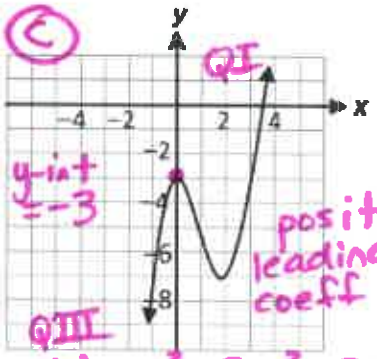


y-int = −3

$h(x) = (x+3)(x-1)$

x-intercepts match

c) C



QI

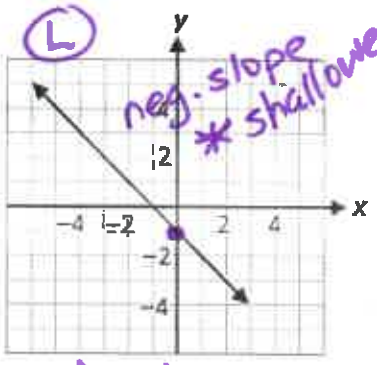
y-int = −3

positive leading coeff.

QIII

$q(x) = x^3 - 3x^2 - 3$

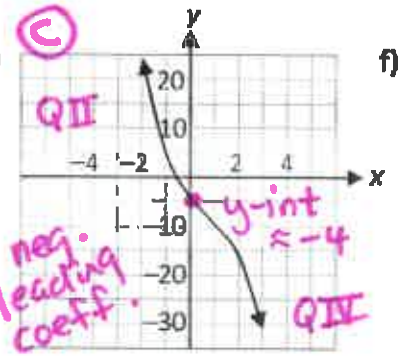
d) L



neg. slope * shallower

$p(x) = -1 - x$

e) C



QII

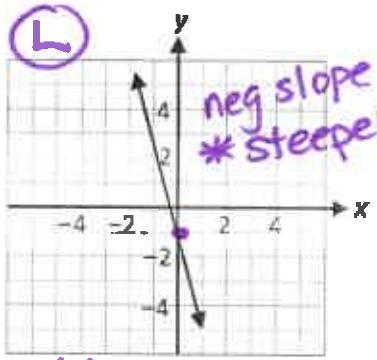
y-int ≈ −4

neg. leading coeff.

QIV

$g(x) = -x^3 + 2x^2 - 6x - 4$

f) L



neg slope * steeper

$K(x) = -4x - 1$

Example: Sketch the graph of a possible polynomial function for each set of characteristics below.

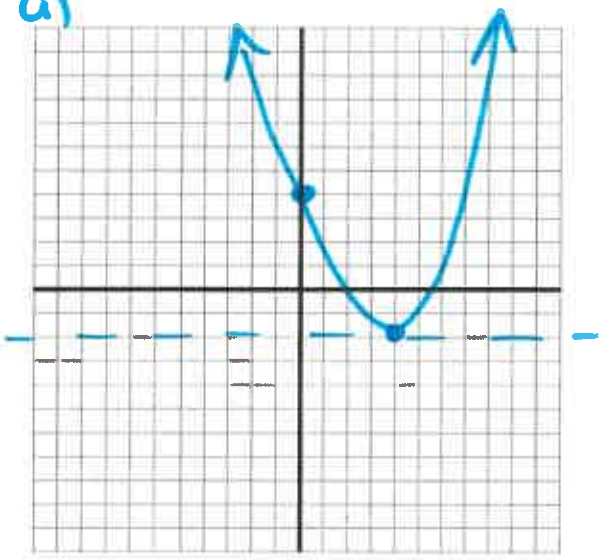
a) Range: $\{y|y \geq -2, y \in R\}$ and a y-intercept of 4.

↳ quadratic, opens up (≥ -2), y-int = 4

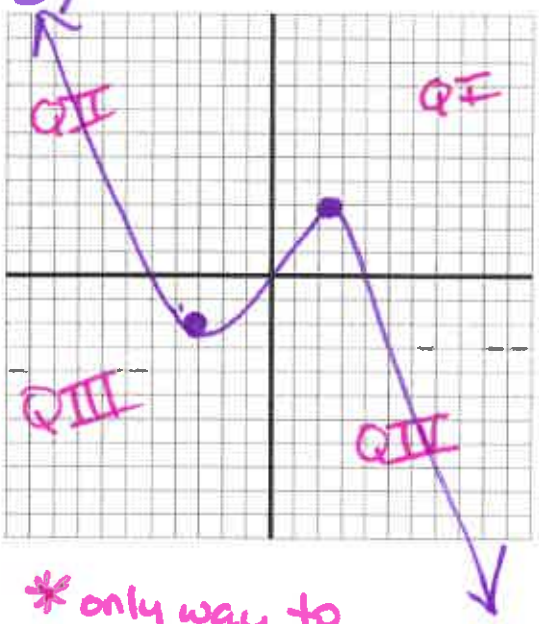
b) Range: $\{y|y \in R\}$, with a turning point in quadrant III and another in quadrant I.

↳ cubic

a)



b)



* only way to have turning points in QIII and QI is to extend from QII to QIV

Key Ideas:

- Max number of x-intercepts equals the degree of the function
- Max number of turning points equals one less than the degree of function
- The constant term in the equation of a polynomial function is the y-intercept of the graph