6.1 – Exploring the Graphs of Polynomial Functions

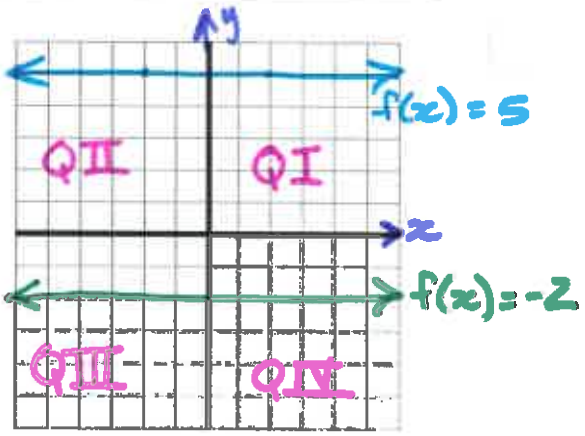
Date: Apr. 13

Definitions:

- **Polynomial function:** A function of one variable that contains only the operations of multiplication and addition, with real-number coefficients and whole number exponents. $f(x) = 5x^3 + 6x^2 - 3x + 7$
- **Degree:** The degree of a polynomial is the highest exponent and determines the shape of the function.
- **End behaviour:** The description of the shape of the graph, from left to right, on the coordinate plane.
- **Turning point:** Any point where the graph of a function changes from increasing to decreasing or decreasing to increasing. (peak ∧, trough ∨)
- **Cubic functions:** A polynomial function of the third degree, whose greatest exponent is three. For example:

Type of function: constant

Sketch:



Examples: $f(x) = 5 \cdot x^0$ $f(x) = -2$

Degree: 0

Number of x-intercepts: none (except $f(x)=0$)

Number of y-intercepts: one

End behaviour: line extends from QII to QI or from QIII to QIV

all possible input values (x)

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y = \text{constant}, y \in R\}$

Range for $f(x)=5$: $\{y \mid y=5, y \in R\}$

all possible output values (y)

Number of turning points: 0

* none of the sketches below represent the example function.

Type of function: cubic

Example: $f(x) = 5x^3 - 2x^2 + x - 1$

Degree: 3

Number of x-intercepts: 1, 2 or 3

Number of y-intercepts: 1

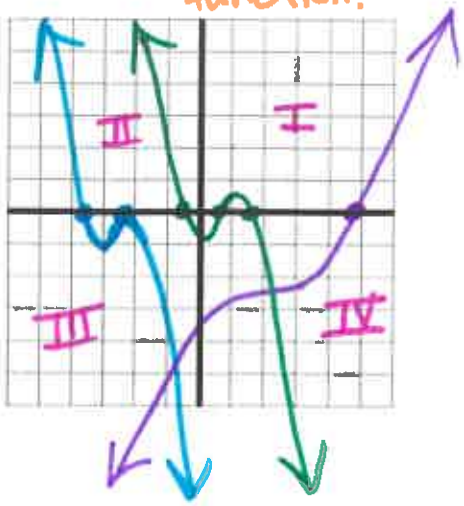
End behaviour: curve extends from QIII to QI or from QII to QIV

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \in R\}$

Number of turning points: 0 or 2

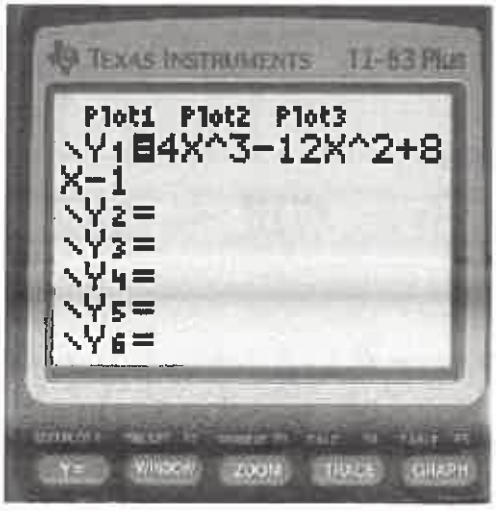
Sketch:

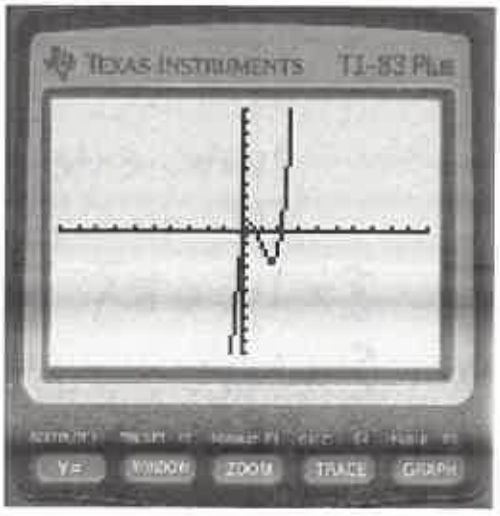


Graphing Functions on the Calculator

To input a function in the TI-83 Calculator, press the Y= button. Use Clear to erase any previous information.

Type the function in using the ^ key to indicate an exponent. Use the X,T,Θ,n for the variable.





To view the graph, press the Graph button. If the graph is not on the screen, adjust the view using the Window button and setting new boundaries.

Type of function: linear

Examples: $f(x)=-2x+1$ $f(x)=x-2$

Degree: 1

Number of x-intercepts: one

Number of y-intercepts: one

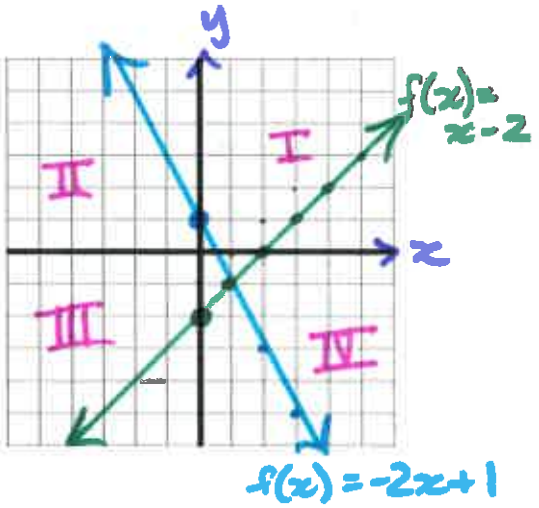
End behaviour: line extends from QIII to QI or from QII to QIV

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \in R\}$

Number of turning points: 0

Sketch:



Type of function: quadratic

Example: $f(x)=2x^2-4$

Degree: 2

Number of x-intercepts: 0, 1 or 2

Number of y-intercepts: 1

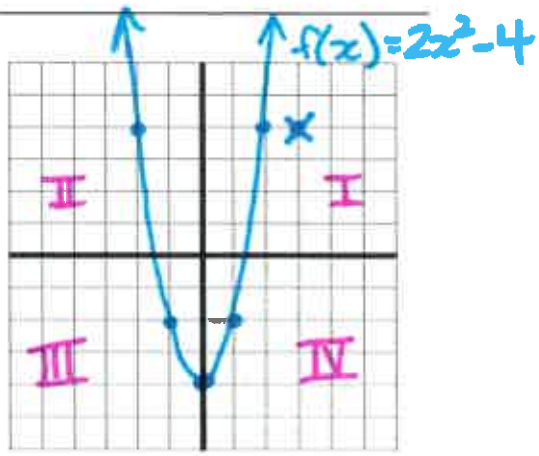
End behaviour: curve extends from QII to QI or from QIII to QIV

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq \text{minimum}, y \in R\}$ or $\{y \mid y \leq \text{maximum}, y \in R\}$

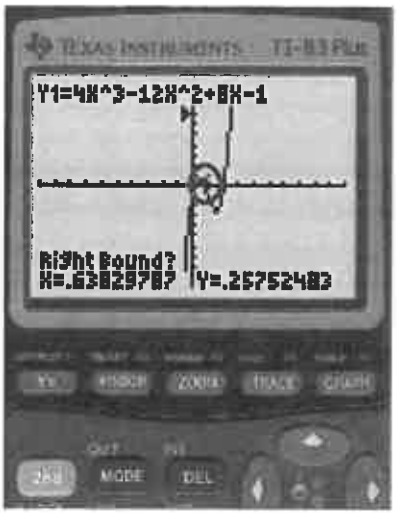
Number of turning points: 1

Sketch:



To solve for an x-Intercept:

- Graph the graph.
- Press 2nd, followed by Trace (Calc).
- In the menu, select 2:zero.
- Select a Left Boundary by moving the cursor to the *left* of the point you suspect is a zero. Press Enter.
- Select a Right Boundary by moving the cursor to the *right* of the point you suspect is a zero. Press Enter.
- Move the cursor close to the solution. (If there are multiple solutions, the calculator will give only the closest. If there are no solutions, the calculator will give ERR: NO SIGN CHNG.)



To solve for a turning point:

- Press 2nd, followed by Trace (Calc).
- You will have to determine if the turning point is a maximum or a minimum by inspecting the graph. Select the appropriate option (3:minimum or 4:maximum).
- Set your boundaries same as for a zero, and put the marker near the max/min before pressing enter.

To solve for the y-Intercept:

- Press 2nd, followed by Trace (Calc).
- Then select 1:value, and enter 0 for x. This will give the y-value when x = 0.

Example: Graph the following:

$$f(x) = 4x^3 - 12x^2 + 8x - 1$$

Determine the following characteristics for the above function:

a) x-intercepts "zero" (0.162, 0) (0.730, 0) (2.107, 0)

b) y-intercepts "value" $x = 0$ (0, −1)

c) end behaviour QIII to QI

d) domain $\{x \mid x \in \mathbb{R}\}$

e) range $\{y \mid y \in \mathbb{R}\}$

f) number of turning points 2 ← @ (0.423, 0.540) "max" and (1.577, −2.540) "min"

g) sketch