5.6 – Independent Events

Date: Apr. 9

Definitions

- **Independent events:** Events whose outcomes are unaffected by each other. The probability of event B occurring does not depend on the probability of event A occurring.

Determining the probability of independent events:

Example: The Spruce Kings sell 11,000 tickets for their Show Home Lottery. There is also a draw for a secondary prize. After the draw for the secondary prize, the winning ticket is re-entered in the draw. If you purchase ten tickets, what is the probability you will win at least one prize?

a. Solve using a Venn Diagram

$P(S) = \frac{10}{11000} = \frac{1}{1100}$ $\qquad P(M) = \frac{1}{1100}$

$P(S') = \frac{1099}{1100}$ $\qquad P(M') = \frac{1099}{1100}$

neither

$P(S \cup M)' = P(S') \cdot P(M')$

$= \frac{1099}{1100} \times \frac{1099}{1100}$

$= 0.99818$

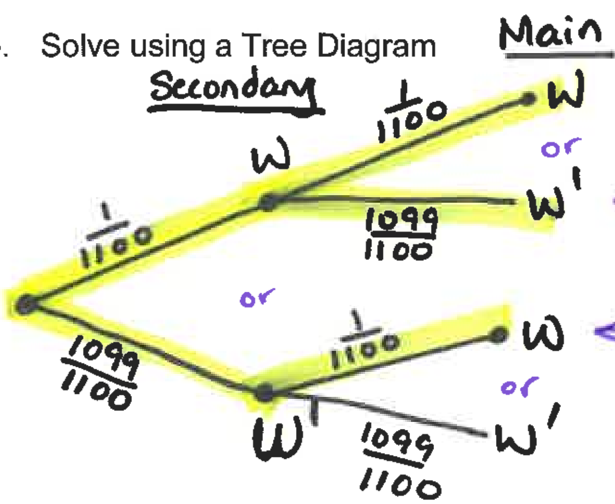
at least one

$P(S \cup M) = 1 - 0.99818$

$= 0.0018$

$= \boxed{0.18\%}$

b. Solve using a Tree Diagram



$\leftarrow P(W \cap W) = \frac{1}{1100} \cdot \frac{1}{1100} = \frac{1}{1100^2}$

$+$

$\leftarrow P(W \cap W') = \frac{1}{1100} \cdot \frac{1099}{1100} = \frac{1099}{1100^2}$

$+$

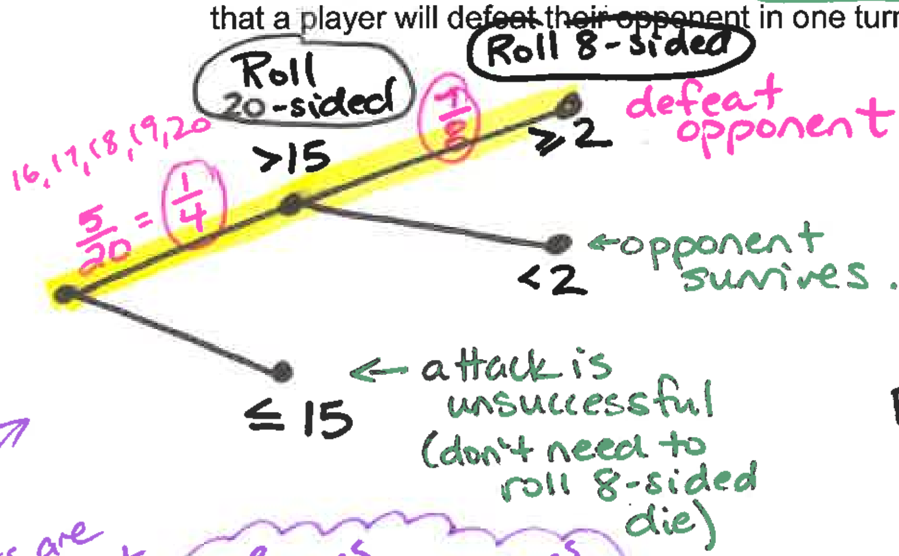
$\leftarrow P(W' \cap W) = \frac{1099}{1100} \cdot \frac{1}{1100} = \frac{1099}{1100^2}$

$P(\text{at least one!}) = \frac{1 + 1099 + 1099}{1100^2}$

$= \frac{2199}{1210000}$

$= 0.0018 \rightarrow \boxed{0.18\%}$

Example: In the popular role-playing game Dungeons and Dragons, players roll a 20 sided die to determine their chance of succeeding at an attack, and then roll an eight-sided die to determine the damage. If a roll over 15 will succeed at the attack, and the opponent only has two hit points, determine the likelihood that a player will defeat their opponent in one turn.



$$P(\text{defeat}) = \frac{1}{4} \times \frac{7}{8} = \boxed{\frac{7}{32}}$$

or $\boxed{21.9\%}$

these Events are independent

(what we roll on the 20-sided die determines whether or not we need to roll the 8-sided die BUT does not change the probability of rolling something ≥2 on an 8-sided die)

INDEPENDENT

$P(A)=0.4$
$P(B)=0.5$
$P(A \cap B)=0.2$
$0.4 \times 0.5 = 0.2$

DEPENDENT

$P(A)=0.4$
$P(B)=0.5$ ← probability of event B changed.
$P(A \cap B)=0.16$
$0.4 \times 0.5 \neq 0.16$

$P(B|A) \neq P(B)$

For **independent events**, the probability that both will occur can be found using the equation:

$$P(A \cap B) = P(A) \times P(B)$$

For **dependent events**, the probability that both will occur is calculated as follows:

$$P(A \cap B) = P(A) \times P(B|A)$$