5.5 – Conditional Probability

(version 2.0)

Date: APR. 5 + 8

Definitions

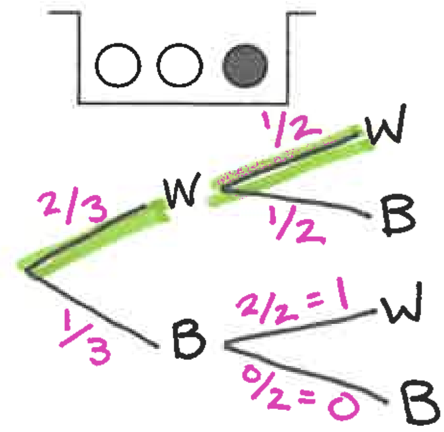
- **Dependent events:** Events whose outcomes are affected by each other.
- **Conditional probability:** The probability of an event occurring given that another event has already occurred.

Calculating the probability of two events:

DEPENDENT

Situation #1: Drawing two balls from the pot, *WITHOUT* replacement.

A ball is randomly selected from the pot and **is not** replaced.
Then a second ball is drawn.
Determine the probability that both balls are white.



- Event A: The first ball is white

  P(A) = $\frac{2}{3}$

- Event B: The second ball is white

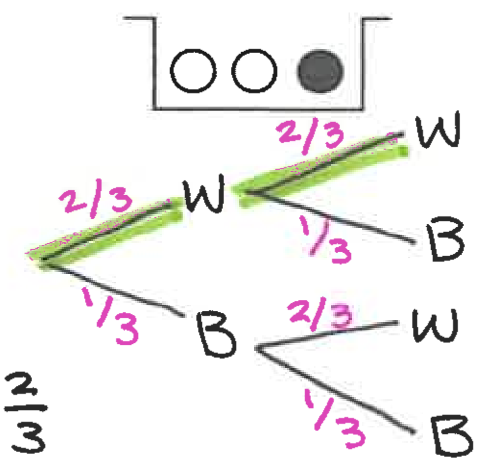
  P(B given that the first ball drawn is white) = $\frac{1}{2}$

P(A and B) = $\frac{\cancel{2}^{1}}{3} \cdot \frac{1}{\cancel{2}_{1}} = \boxed{\frac{1}{3}}$

INDEPENDENT

Situation #2: Drawing two balls from the pot, *WITH* replacement.

A ball is randomly selected from the pot and **is** replaced.
Then a second ball is drawn.
Determine the probability that both balls are white.



- Event A: The first ball is white

  P(A) = $\frac{2}{3}$

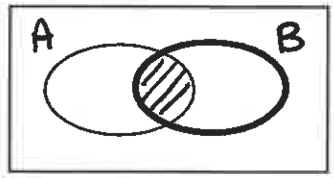
- Event B: The second ball is white

  P(B given that the first ball drawn is white) = $\frac{2}{3}$

P(A and B) = $\frac{2}{3} \cdot \frac{2}{3} = \boxed{\frac{4}{9}}$

Probability of Events A and B

$$P(A \cap B) = P(A) \cdot P(B|A)$$



Conditional Probability ("probability of B given A")

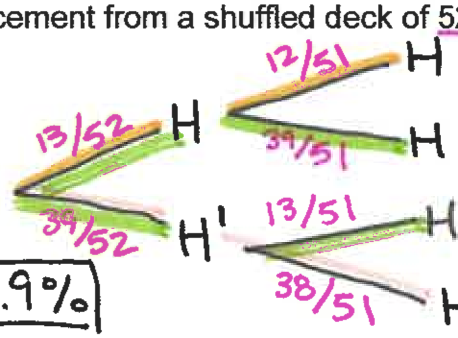
$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Note: Events A and B are <u>independent</u> if $P(B|A) = P(B)$

For independent events: $P(A \cap B) = P(A) \cdot P(B)$

H = heart (13)   H' = not heart (39)

Example: Two cards are drawn without replacement from a shuffled deck of 52 cards.
What is the probability that:



a. both cards are hearts?

$$P(H \cap H) = \frac{\cancel{13}^{1}}{\cancel{52}_{4}} \cdot \frac{\cancel{12}^{\cancel{3}\,1}}{\cancel{51}_{17}}$$

$$= \boxed{\frac{1}{17}} \text{ or } \boxed{0.0588} \text{ or } \boxed{5.9\%}$$

b. neither card is a heart?

$$P(H' \cap H') = \frac{\cancel{39}^{13}}{\cancel{52}_{4}} \cdot \frac{\cancel{38}^{19}}{\cancel{51}_{17}}$$

$$= \boxed{\frac{19}{34}} \text{ or } \boxed{0.5588} \text{ or } \boxed{55.9\%}$$

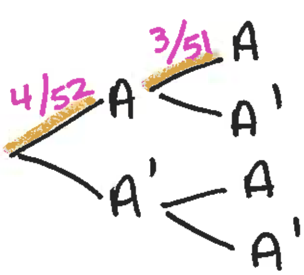
c. exactly one of the two cards is a heart?

$$P(H \cap H') \text{ or } P(H' \cap H) = \frac{\cancel{13}^{1}}{\cancel{52}_{4}} \cdot \frac{39^{13}}{\cancel{51}_{17}} + \frac{\cancel{39}^{\cancel{3}\,1}}{\cancel{52}_{4}} \cdot \frac{13}{\cancel{51}_{17}}$$

$$= \frac{13}{68} + \frac{13}{68}$$

$$= \frac{26}{68} \longrightarrow \boxed{\frac{13}{34}} \text{ or } \boxed{0.3824} \text{ or } \boxed{38.2\%}$$

d. both cards are aces?



$$P(A \cap A) = \frac{\cancel{4}^{1}}{\cancel{52}_{13}} \cdot \frac{\cancel{3}^{1}}{\cancel{51}_{17}}$$

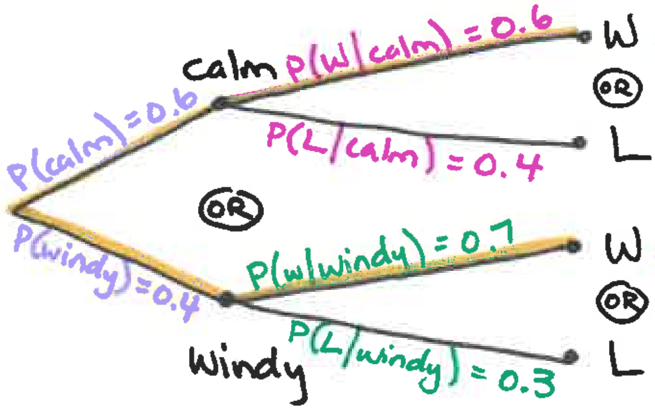
$$= \boxed{\frac{1}{221}} \text{ or } \boxed{0.0045} \text{ or } \boxed{0.5\%}$$

$P(W|calm) = 0.6$ $P(W|windy) = 0.7$

Example: Hillary is the coach of a junior ultimate team. Based on the team's record, they have a 60% chance of winning on calm days and a 70% chance of winning on windy days. Tomorrow, there is a 40% chance of high winds. Assuming there are no ties, what is the probability that Hillary's team will win tomorrow?

$P(windy) = 0.4$
$P(calm) = 1 - 0.4 = 0.6$

weather AND game result



$P(calm \cap W) = P(calm) \cdot P(W|calm) = 0.6 \times 0.6 = 0.36$

$+$

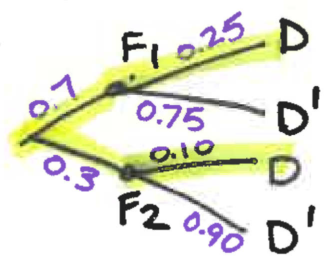
$P(windy \cap W) = P(windy) \cdot P(W|windy) = 0.4 \times 0.7 = 0.28$

$$P(W) = P(calm \cap W) + P(windy \cap W) = 0.36 + 0.28 = 0.64$$

$\boxed{64\%}$ probability of win

Example: A company has two factories that make computer chips. Suppose 70% of the chips come from Factory 1 and 30% come from Factory 2. In Factory 1, 25% of the chips are defective; in Factory 2, 10% of the chips are defective.

a. Suppose it is not known from which factory a chip came. What is the probability that the chip is defective?

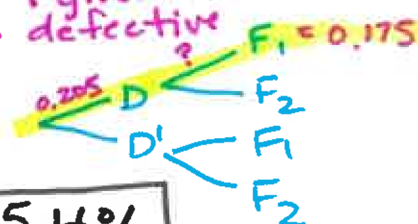


$$P(D) = P(F_1 \cap D) + P(F_2 \cap D)$$
$$= P(F_1) \cdot P(D|F_1) + P(F_2) \cdot P(D|F_2)$$
$$= (0.7)(0.25) + (0.3)(0.1)$$
$$= 0.175 + 0.03$$
$$= 0.205$$

$\boxed{20.5\%}$

b. Suppose a defective chip is discovered. What is the probability that the chip came from Factory 1?

want to find $P(F_1|D)$ ← probability it came from Factory 1 given that it is defective



$$P(F_1|D) = \frac{P(F_1 \cap D)}{P(D)}$$

$$= \frac{0.175}{0.205} = 0.8537$$

$\boxed{85.4\%}$

From part a)
$P(F_1 \cap D) = 0.175$
$P(D) = 0.205$