5.5 – Conditional Probability

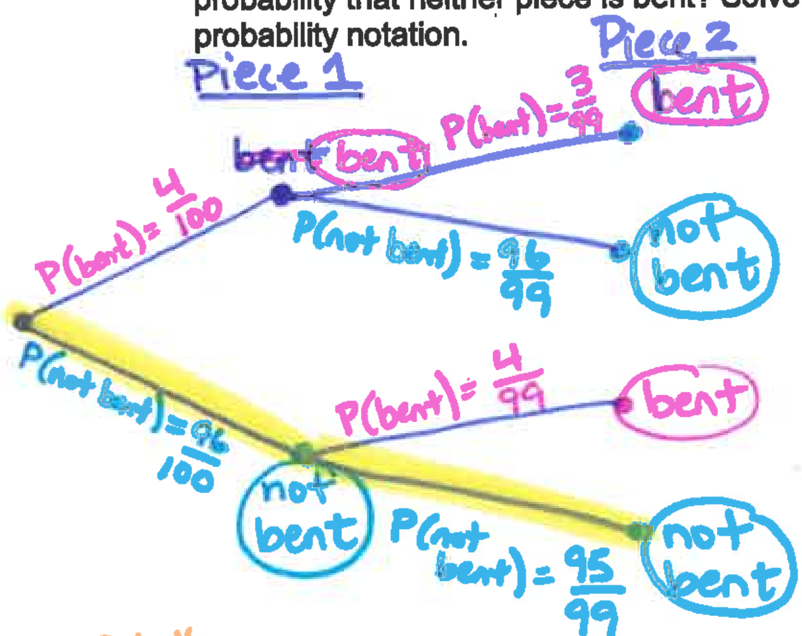
Date: Apr. 3

Definitions

- **Dependent events:** Events whose outcomes are affected. by each other.
- **Conditional probability:** The probability of an event occurring given that another event has already occurred.

Calculating the probability of two events:

Example: An installer for drop ceilings knows that every box of 100 typically has 4 bent pieces. The installer draws two pieces at random from a box. What is the probability that neither piece is bent? Solve using a tree diagram and using probability notation.



draw piece 1 AND draw piece 2

$$P(\text{not bent, not bent}) = \frac{96}{100} \times \frac{95}{99}$$

$$= \frac{9120}{9900} \begin{matrix} \div 60 \\ \div 60 \end{matrix}$$

$$= \boxed{\frac{152}{165}}$$

or $\boxed{92.1\%}$

probability of both events A and B

probability of A?

$$\boxed{P(A \cap B) = P(A) \cdot P(B|A)}$$

probability of event B given that event A has happened.

Conditional probability can also be calculated using the formula:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

same formula as above (rearranged)

$U = \{1, 2, 3 \dots 40\}$ $n(U) = 40$

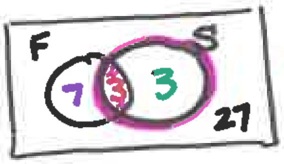
inclusive

Example: Find the probability that a number between 1 and 40 is a multiple of four given that it is a multiple of six. Solve using a Venn Diagram and using a formula.

$F = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40\}$ $n(F) = 10$

$S = \{6, 12, 18, 24, 30, 36\}$ $n(S) = 6$

$n(F \cap S) = 3$



From Diagram

→ we have 6 multiples of six

→ 3 of those are multiples of four

$P(F|S) = \frac{3}{6}$

$= \boxed{\frac{1}{2}}$

Formula

$P(A \cap B) = P(A) \cdot P(B|A)$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

$P(F|S) = \frac{P(F \cap S)}{P(S)}$ ← $3/40$, ← $6/40$

$= \frac{3}{40} \div \frac{6}{40}$

$= \frac{3}{\cancel{40}} \times \frac{\cancel{40}}{6} = \frac{3}{6} = \boxed{\frac{1}{2}}$

Example: According to medical records, a test for a particular lung disease is accurate 94% of the time, and 6.4% of the population actually has the lung disease. Determine the probability that:

$100 - 6.4 = 93.6\%$

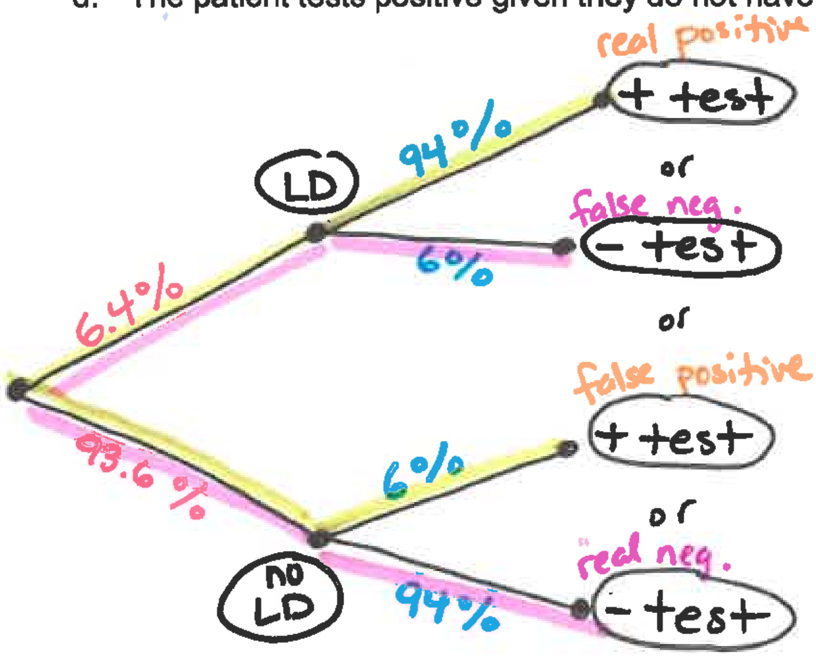
$100 - 94 = 6\%$

a. The patient tests positive.

b. The patient tests negative.

c. The patient tests positive if it is known that they have the disease.

d. The patient tests positive given they do not have the lung disease.



a) $P(+\text{test})$

$= (6.4\%)(94\%) + (93.6\%)(6\%)$

$= (0.064)(0.94) + (0.936)(0.06)$

$= 0.11632 \rightarrow \boxed{11.6\%}$

b) $P(-\text{test})$

$= (0.064)(0.06) + (0.936)(0.94)$

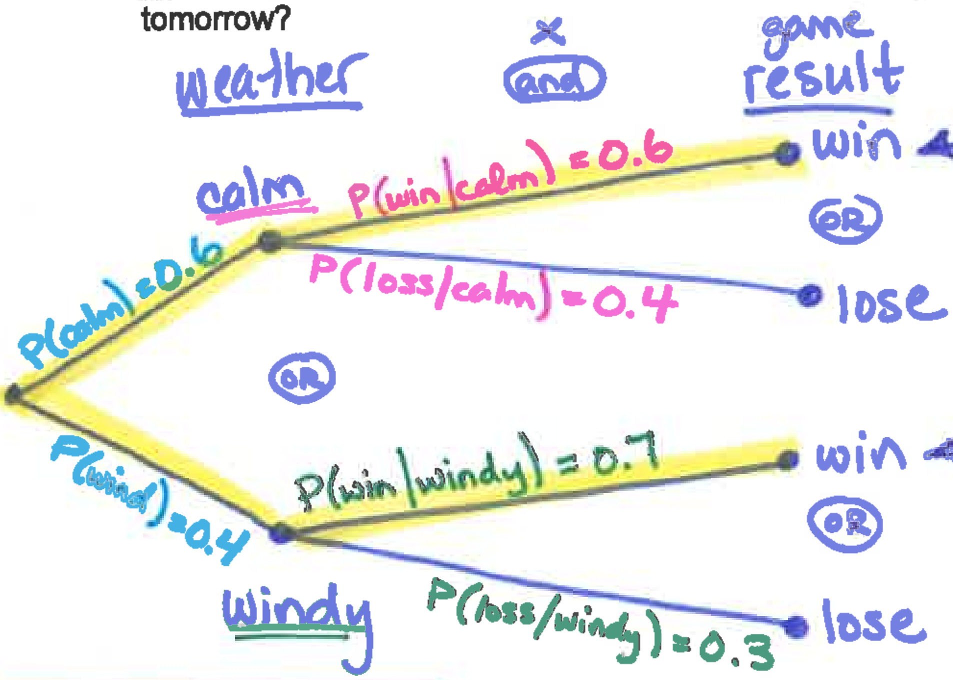
$= 0.88368 \rightarrow \boxed{88.4\%}$

c) $P(+\text{test} \mid LD) = \boxed{94\%}$

d) $P(+\text{test} \mid \text{no } LD) = \boxed{6\%}$

Example: Hillary is the coach of a junior ultimate team. Based on the team's record, they have a 60% chance of winning on calm days and a 70% change of winning on windy days. Tomorrow, there is a 40% chance of high winds. Assuming there are no ties, what is the probability that Hillary's team will win tomorrow?

$40\% = 0.4$



$P(\text{calm}, \text{win}) = 0.6 \times 0.6 = 0.36$

$+$

$P(\text{windy}, \text{win}) = 0.4 \times 0.7 = 0.28$

USING FORMULA:

$P(\text{windy}) = 40\% = 0.4$

$P(\text{calm}) = 60\% = 0.6$

$P(\text{win}|\text{windy}) = 70\% = 0.7$

$P(\text{win}|\text{calm}) = 60\% = 0.6$

$P(\text{win}) = 0.36 + 0.28 = 0.64$

$\boxed{\text{64\% probability of win}}$

$$P(\text{win}) = P(\text{windy} \cap \text{win}) + P(\text{calm} \cap \text{win})$$
$$= P(\text{windy}) \cdot P(\text{win}|\text{windy}) + P(\text{calm}) \cdot P(\text{win}|\text{calm})$$
$$= (0.4)(0.7) + (0.6)(0.6)$$
$$= 0.28 + 0.36$$
$$= 0.64 \longrightarrow \boxed{64\%}$$