5.3 – Probabilities Using Counting Methods

Date: Mar. 29

Solving a probability problem using counting methods:

$$P = \frac{\#\text{ fav. outcomes}}{\text{total \# of outcomes}}$$

Example: Jamaal, and Ethan enter a rock-paper-scissors competition. 7 students from another class also enter. What is the probability that Jamaal, and Ethan face each other in the final? (ie: be the last two in the competition.) 9 competitors

a. Solve using permutations. (considers order)

#fav. outcomes: $_2P_2 = 2$ (J 1st and E 2nd, or J 2nd and E 1st)

total # of outcomes: $_9P_2 = 72$

$P = \frac{2}{72} = \boxed{\frac{1}{36}}$ or $\boxed{2.8\%}$

b. Solve using combinations. (order doesn't matter)

fav. outcomes: $_2C_2 = 1$ (J+E in final)

total # of outcomes: $_9C_2 = 36$

$P = \boxed{\frac{1}{36}}$ or $\boxed{2.8\%}$

Solving a probability using the Fundamental Counting Principle:

Recall: **Fundamental Counting Principle:** if there are 'a' ways to perform one task, and 'b' ways to perform another, then there are '$a \times b$' ways to perform both.

Example: Yarenda has four pairs of pants, six t-shirts, and eight pairs of socks. One pair of pants is striped, two t-shirts are striped, and six pairs of socks are striped. If Yarenda dresses randomly, what is the probability she will be entirely in stripes?

#fav. outcomes (all stripes): $1 \cdot 2 \cdot 6 = 12$ "all stripes" outfits.

total # of outcomes (any outfit): $4 \cdot 6 \cdot 8 = 192$ outfits total

$P(\text{all stripes}) = \frac{12}{192} = \boxed{\frac{1}{16}}$

or $\boxed{6.25\%}$

What are the odds in favour?

all stripes : NOT all stripes

12 : 192 − 12 (total − fav)

12 : 180

$\boxed{1:15}$

Solving a probability problem using reasoning:

Example: Beau hosts a morning radio show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHEWAN with letter tiles. Then he turns the tiles face down and mixes them up. He asks Sally to arrange the tiles in a row and turn them face up. If the row of tiles spells SASKATCHEWAN, Sally will win a new car. Determine the probability that Sally will win the car.

fav. outcomes: 1
SASKATCHEWAN

total # of possible outcomes:
12 letters
→ 2 S
→ 3 A

$\frac{12!}{2!3!} = 39\,916\,800$

$P = \frac{1}{39\,916\,800}$

Example: Evelyn is walking from her work, at point A, to her house at point B. She chooses to go East or South randomly at each intersection. What is the probability of her passing by the grocery store at C?

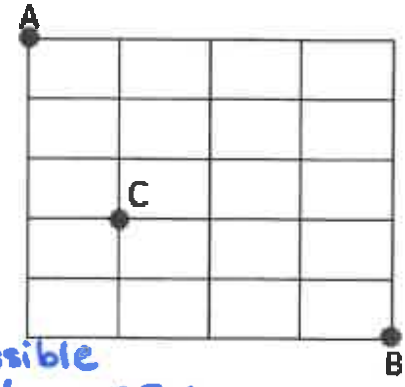
fav. outcomes
(walks past store)

A → C and C → B

$= \frac{4!}{1!3!} \cdot \frac{5!}{3!2!}$

$= 4 \cdot 10$

$= 40$



total # of possible outcomes:
A → B $= \frac{9!}{4!5!}$
$= 126$

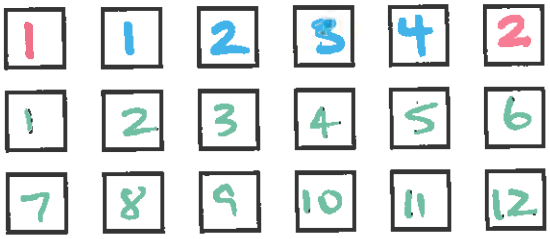
[illegible]

$P = \frac{40}{126} = \frac{20}{63}$ or 31.7%

* assume that class is full

Example: There are 18 bikes in Marnie's spinning class. The bikes are arranged in 3 rows, with 6 bikes in each row. Allison, Brett, Carol, Doug, Erica, and Franco each call the gym to reserve a bike. They hope to be in the same row, but they cannot request a specific bike. Determine the probability that all friends will be in the same row, with Allison and Franco at either end.

$P = \frac{\text{fav}}{\text{total}}$



- Number of ways to seat Allison and Franco at either end: of one row

  A - - - - F F - - - - A $_2P_2 = 2!$

- Number of ways to seat the other 4 friends:

  - - - - $_4P_4 = 4!$

- Number of ways to seat the other 12 people in the class:

  $_{12}P_{12} = 12!$

- Number of ways to seat the 6 friends in Row 1:

  Seat friends AND seat rest of friends AND seat rest of class

  $2! \cdot 4! \cdot 12!$

- Number of ways to seat the 6 friends in ANY row:

  Row 1 OR Row 2 OR Row 3

  $2!\cdot4!12! + 2!4!12! + 2!4!12!$ → # fav. options $3 \cdot 2! \cdot 4! \, 12!$

- Total number of ways to ~~assign 18 people to 18 bikes~~:

  total

  $_{18}P_{18}$ OR $18!$ total # of possible outcomes.

$$P = \frac{3 \cdot 2! \cdot 4! \cdot 12!}{18!}$$

$$= \frac{3 \cdot 2! \cdot 4! \cdot \cancel{12!}}{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot \cancel{12!}}$$

$$= \frac{\cancel{3} \cdot \cancel{2} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{18} \cdot 17 \cdot \cancel{16} \cdot 15 \cdot 14 \cdot 13}$$

$$= \frac{4 \cdot 2}{17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}$$

$$= \frac{1}{17 \cdot 2 \cdot 15 \cdot 14 \cdot 13}$$

$$= \frac{1}{92\,820}$$