5.2 – Probability and Odds

comparison of favourable and unfavourable outcomes

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Definitions

- **Odds in favour:** The ratio of the probability that an event will occur to the probability that the event will not occur. $\boxed{\text{fav : unfav}}$

  (The ratio of the number of favourable outcomes to the number of unfavourable outcomes.)

- **Odds against:** The ratio of the probability that an event will not occur to the probability that the event will occur. $\boxed{\text{unfav : fav}}$

  (The ratio of the number of unfavourable outcomes to the number of favourable outcomes.)

Determining odds using sets:

$n(T) + n(T') = n(U)$

Example: Carmen is rolling a twenty-sided die. What are the odds of rolling a multiple of 3?

universal set → U = {1, 2, 3, 4 ... 19, 20} $n(U) = 20$

multiples of 3 → T = {3, 6, 9, 12, 15, 18} $n(T) = 6$ favourable outcomes

everything else (not a multiple of 3) → T' = {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20} $n(T') = 14$ unfav. outcomes

Odds in favour of rolling a multiple of three:

Odds of (T)

$6 : 14$ ÷2 → $\boxed{3 : 7}$ or $\frac{3}{7}$

Determining odds from probability:

When we have the probability of an event occurring, the probability is given by:

$$P(A) = \frac{n(A)}{n(U)}$$

$n(A)$ ← fav. outcomes

$n(U)$ ← total outcomes.

The total number of outcomes includes all favourable and unfavourable outcomes, so we can subtract the number of favourable outcomes from the total to determine the number of unfavourable outcomes.

unfav. outcomes

$$n(A') = n(U) - n(A)$$

odds in favour	odds against
$n(A) : n(A')$	$n(A') : n(A)$
$n(A) : n(U) - n(A)$	

Example: The Canucks have a 64% probability of beating the Islanders when playing a home game. What are the odds the Canucks will win when they play the Islanders at home? What are the odds against a Canucks win?

$P(win) = 64\%$
$= \frac{64}{100}$

$P(\text{not win}) = 100 - 64$
$= 36\%$

Odds(win)

$P(win) : P(\text{not win})$

$\div 4 \; 64 : 36 \; \div 4$

$\boxed{16 : 9}$

odds against (win) = odds(not win)

$\boxed{9 : 16}$

Determining probability from odds:

Example: The odds of winning the the roll-up-the-rim contest in 2019 were 10 : 27.

win → 10, win' → 27

a. What was the probability of winning on any particular cup?

$n(w) = 10$
$n(w') = 27$

$n(U) = 10 + 27$
$= 37$

$P(w) = \boxed{\frac{10}{37}}$ OR $\boxed{0.27}$ OR $\boxed{27\%}$

fav. outcomes (win)

total possible outcomes

b. How many winning cups would you expect if you purchased two coffees a day for ~~January~~?
March?

31 days in March × 2 = 62 cups of coffee

$\frac{10}{37} = \frac{x}{62}$

$x = \frac{10 \times 62}{37}$
$= 16.76$ →

$\boxed{\text{Expect about 17 winning cups}}$

Making a decision based on odds and probability:

Example:

a. Which is more likely to successfully roll an 8: using an 8 sided die, or using two 4 sided dice? Use odds to compare your answer.

8 sided die

1, 2, 3, 4, 5, 6, 7, 8

odds of rolling 8

$\boxed{1 : 7}$

fav unfav

Two 4-sided die

16 possible outcomes

$\boxed{1 : 15}$

fav unfav.

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

8-sided die is more likely to roll an 8

* has less unfav. outcomes for the same number of fav. outcomes

b. What if you want to roll a ~~sum~~ total greater than 4?

8-sided die

5, 6, 7, 8 1, 2, 3, 4

4 : 4

fav. unfav.

$\boxed{1 : 1}$

Two 4-sided die

10 : 6

fav unfav

$\boxed{5 : 3}$

to compare

① use logic (two 4-sided die have more fav. than unfav. outcomes)

Two 4-side dice are more likely to roll a total > 4

OR ② andjust ratios so they have a common value.

e.g. compare 3:3 to 5:3