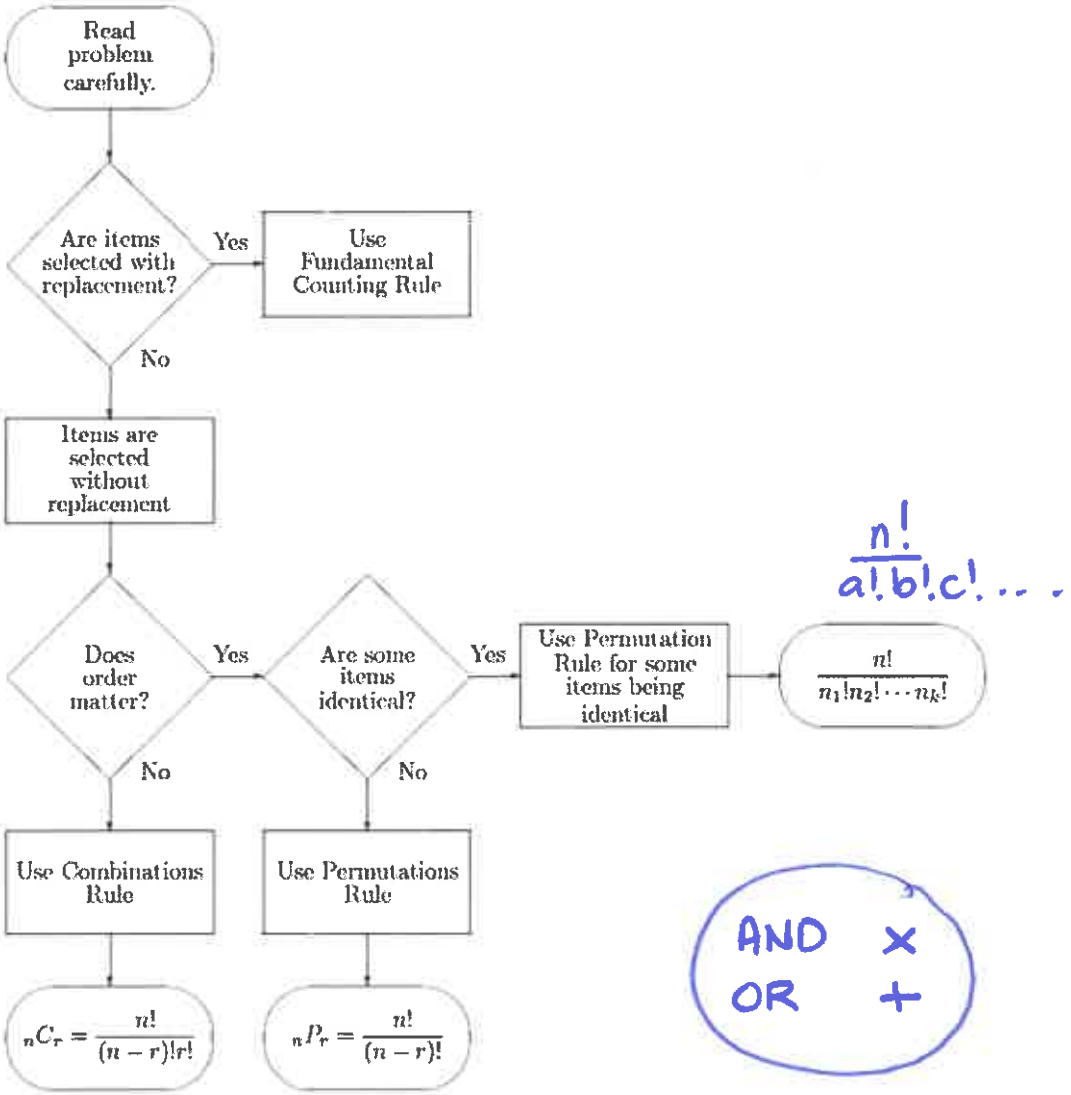
4.7 – Solving Counting Problems

Date: Mar. 4

Steps for solving counting problems:

1. **Read the question carefully!**
2. **Determine if order plays a role in the situation.**
   - If order matters → use permutation formula
   - If order does not matter → use combination formula
3. **Look for conditions and consider these in your solution.**
   - If a problem involves multiple tasks that are connected by the word AND, then the Fundamental Counting Principle can be applied → MULTIPLY the number of ways that each task can occur.
   - If a problem involves multiple tasks that are connected by the work OR, then the Fundamental Counting Principle does not apply → ADD the number of ways that each task can occur (this typically is found in counting problems that involve several cases).



Example: At a family reunion, there are two grandparents, four parents and six children. If the photographer wants the family grouped by generations, how many different possible photos are there?

* order matters! PERMUTATION

arrange members of each generation:

GRANDPARENTS: $\frac{2}{} \times \frac{1}{} = 2!$ or ${}_2P_2$

PARENTS: $\frac{4}{} \times \frac{3}{} \times \frac{2}{} \times \frac{1}{} = 4!$ or ${}_4P_4$

KIDS: $= 6! = {}_6P_6$

AND

arrange generations: $3!$ or ${}_3P_3$

G P K
K G P
⋮

arrange grandparents, arrange parents, arrange kids, arrange generations

$${}_2P_2 \times {}_4P_4 \times {}_6P_6 \times {}_3P_3$$
$$= 2(24)(720)(6)$$
$$= \boxed{207\,360} \text{ possible photos.}$$

Example: The same photographer wants another photo with one grandparent on each end and the parents in the middle. How many different photos are possible?

$$\frac{2}{G} \times \frac{6}{K} \times \frac{5}{K} \times \frac{4}{K} \times \frac{4}{P} \times \frac{3}{P} \times \frac{2}{P} \times \frac{1}{P} \times \frac{3}{K} \times \frac{2}{K} \times \frac{1}{K} \times \frac{1}{G}$$

$$2! \times 4! \times 6!$$

OR $${}_2P_2 \times {}_4P_4 \times {}_6P_6$$

$$= 2(24)(720) = \boxed{34\,560} \text{ possible photos.}$$

Example: A class trip of 12 students is booked into a hotel. The school booked one room for 4 students, two rooms for 3 students, and one room for 2 students. How many different ways can the students be arranged into the rooms?

* order does not matter COMBINATION

ROOM 1	ROOM 2	ROOM 3	ROOM 4
4	3	3	2

fill room 1 → room 2 → room 3 → room 4

$${}_{12}C_4 \times {}_8C_3 \times {}_5C_3 \times {}_2C_2$$

8 left, 5 left, 2 left

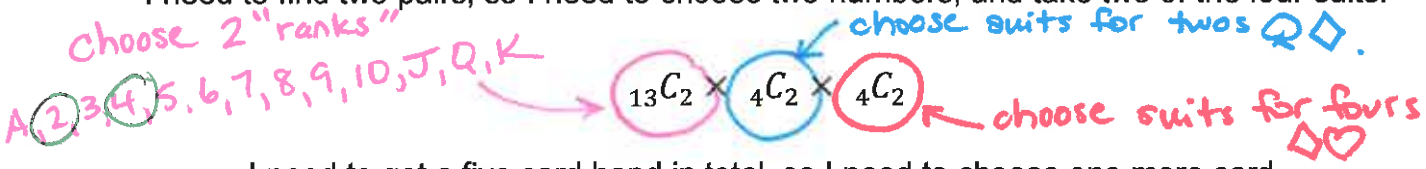
$$= (495)(56)(10)(1) = \boxed{277\,200} \text{ different ways.}$$

Example: Inigo was asked to find how many ways to pull two pairs in a five-card poker hand. Inigo did the following calculation. Find the error in Inigo's work and find the correct number of ways to get two pairs.

Example Hand.

Inigo's Work

I need to find two pairs, so I need to choose two numbers, and take two of the four suits.



I need to get a five card hand in total, so I need to choose one more card.

Four cards have been removed from the deck.

ERROR

48 cards remaining contain:

2♡ 2♧ 4♤ 4♧

My result will be:

$$_{13}C_2 \times {}_4C_2 \times {}_4C_2 \times {}_{48}C_1$$

44

We will have a full house if we pick one of these!

We only have 44 cards to choose from that give us two pairs.

$= {}_{13}C_2 \times {}_4C_2 \times {}_4C_2 \times {}_{44}C_1$

$= 78(6)(6)(44)$

$= 123\,552$ ways to pull two pairs.