

4.6 – Combinations

Date: Mar. 2

when order matters	Permutations	nP_r	$nP_r = \frac{n!}{(n-r)!}$
when order DOES NOT matter	Combinations	nC_r or $\binom{n}{r}$	$nC_r = \frac{n!}{(n-r)!r!}$

Restrictions can complicate problems involving combinations.

Example: There are 7 dogs and 8 cats in a kennel. How many ways can four animals be taken out for showing if there must be at least one dog and one cat? Solve two ways.

option 1
DIRECT REASONING

FOUR ANIMALS

$$\begin{aligned} &\rightarrow 2 \text{ cats, } 2 \text{ dogs} \rightarrow 8C_2 \times 7C_2 = 28 \times 21 = 588 \\ &\text{OR} \\ &\rightarrow 1 \text{ cat, } 3 \text{ dogs} \rightarrow 8C_1 \times 7C_3 = 8 \times 35 = 280 \\ &\text{OR} \\ &\rightarrow 3 \text{ cats, } 1 \text{ dog} \rightarrow 8C_3 \times 7C_1 = 56 \times 7 = 392 \end{aligned}$$

1260 ways

option 2
INDIRECT REASONING

total \rightarrow no cat no dog

$$15C_4 - 8C_0 \times 7C_4 - 8C_4 \times 7C_0$$

$$1365 - 1(35) - 70(1) = \mathbf{1260} \text{ ways}$$

Example: Determine the number of ways that a committee of 5 people can be made from 10 adults and 6 youth, given that there must be at least 1 youth on the committee.

DIRECT REASONING.

$$\begin{aligned} 1Y, 4A: & 6C_1 \cdot 10C_4 = 6(210) = 1260 \\ 2Y, 3A: & 6C_2 \cdot 10C_3 = 15(120) = 1800 \\ 3Y, 2A: & 6C_3 \cdot 10C_2 = 20(45) = 900 \\ 4Y, 1A: & 6C_4 \cdot 10C_1 = 15(10) = 150 \\ 5Y, 0A: & 6C_5 \cdot 10C_0 = 6(1) = 6 \end{aligned}$$

4116
ways

INDIRECT REASONING.

total \rightarrow no youth / 5 adults

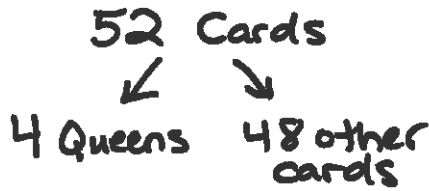
$$16C_5 - 6C_0 \cdot 10C_5$$

$$= 4368 - 1(252)$$

$$= \mathbf{4116} \text{ ways}$$

Assume a standard deck of cards with NO Jokers.

Example: How many ways can five cards be chosen that have at most two Queens?



No Queens: ${}^4C_0 \cdot {}^{48}C_5 = 1\,712\,304$
 OR 1 Queen: ${}^4C_1 \cdot {}^{48}C_4 = +778\,320$
 OR 2 Queens: ${}^4C_2 \cdot {}^{48}C_3 = +103\,776$

2 594 400

Calculations can also be performed using combination notation.

Example: Solve for n.

${}^nC_2 = 36$

$\frac{n!}{(n-2)!2!} = 36$
 $\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}2!} = 36$
 $\frac{n(n-1)}{2} = 36 \cdot 2!$
 $n(n-1) = 36 \cdot 2 \cdot 1$
 $n^2 - n = 72$
 $n^2 - n - 72 = 0$ $\frac{-9 \times 8}{-9 + 8} = -72$
 $(n-9)(n+8) = 0$
 $n-9=0$ OR $n+8=0$
 $n=9$ OR $n=-8$ reject

Example: Solve for n.

$\binom{n}{2} = 6$

$nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$

$\frac{n!}{(n-2)!2!} = 6$
 $\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}2!} = 6$
 $\frac{n(n-1)}{2} = 6 \cdot 2!$
 $n(n-1) = 6 \cdot 2 \cdot 1$
 $n(n-1) = 12$
 $n^2 - n = 12$
 $n^2 - n - 12 = 0$ $\frac{-4 \times 3}{-4 + 3} = -12$
 $(n-4)(n+3) = 0$
 $n-4=0$ OR $n+3=0$
 $n=4$ OR $n=-3$ reject