4.6 – Combinations

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when order matters			
	Permutations	$_nP_r$	$_nP_r = \dfrac{n!}{(n-r)!}$
when order DOES NOT matter	Combinations	$_nC_r$ or $\binom{n}{r}$	$_nC_r = \dfrac{n!}{(n-r)!r!}$

Restrictions can complicate problems involving combinations.

Example: There are 7 dogs and 8 cats in a kennel. How many ways can four animals be taken out for showing if there must be at least one dog and one cat? Solve two ways.

option 1
DIRECT REASONING

FOUR ANIMALS

→ 2 cats, 2 dogs → $_8C_2 \times {_7C_2} = 28 \times 21 = 588$

+

→ 1 cat, 3 dogs → $_8C_1 \times {_7C_3} = 8 \times 35 = 280$

+

→ 3 cats, 1 dog → $_8C_3 \times {_7C_1} = 56 \times 7 = 392$

$\boxed{1260}$ ways

option 2
INDIRECT REASONING

$\underbrace{_{15}C_4}_{\text{total}} - \underbrace{_8C_0 \times {_7C_4}}_{\text{no cat}} - \underbrace{_8C_4 \times {_7C_0}}_{\text{no dog}}$

$1365 - 1(35) - 70(1) = \boxed{1260}$ ways

Example: Determine the number of ways that a committee of 5 people can be made from 10 adults and 6 youth, given that there must be at least 1 youth on the committee.

DIRECT REASONING.

1Y, 4A: $_6C_1 \cdot {_{10}C_4} = 6(210) = 1260$

+

or 2Y, 3A: $_6C_2 \cdot {_{10}C_3} = 15(120) = 1800$

+

or 3Y, 2A: $_6C_3 \cdot {_{10}C_2} = 20(45) = 900$

+

or 4Y, 1A: $_6C_4 \cdot {_{10}C_1} = 15(10) = 150$

+

or 5Y, 0A: $_6C_5 \cdot {_{10}C_0} = 6(1) = 6$

$\boxed{4116}$ ways

INDIRECT REASONING.

$\underbrace{_{16}C_5}_{\text{total}} - \underbrace{_6C_0 \cdot {_{10}C_5}}_{\text{no youth / 5 adults}}$

$= 4368 - 1(252)$

$= \boxed{4116}$ ways

Assume a standard deck of cards with No Jokers.

Example: How many ways can five cards be chosen that have at most two Queens?

52 Cards

4 Queens   48 other cards

No Queens: ${}_4C_0 \cdot {}_{48}C_5 = 1\,712\,304$

or 1 Queen: ${}_4C_1 \cdot {}_{48}C_4 = +778\,320$

or 2 Queens: ${}_4C_2 \cdot {}_{48}C_3 = +103\,776$

$\boxed{2\,594\,400}$

Calculations can also be performed using combination notation.

${}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Example: Solve for n.

$${}_nC_2 = 36$$

$$\frac{n!}{(n-2)!\,2!} = 36$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!} = 36$$

$$\cancel{2!}\cdot\frac{n(n-1)}{\cancel{2!}} = 36\cdot 2!$$

$$n(n-1) = 36\cdot 2\cdot 1$$

$$n^2 - n = \cancel{72}$$
$$-72 \quad \cancel{-72}$$

$$n^2 - 1n - 72 = 0$$

$-9 \times 8 = -72$

$-9 + 8 = -1$

$$(n-9)(n+8) = 0$$

$n - \cancel{9} = 0$ or $n + \cancel{8} = 0$

$+\cancel{9} \quad +9 \qquad -\cancel{8} \quad -8$

$\boxed{n = 9}$ or $\cancel{n = -8}$ reject

Example: Solve for n.

$$\binom{n}{2} = 6$$

$$\frac{n!}{(n-2)!\,2!} = 6$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!} = 6$$

$$\cancel{2!}\cdot\frac{n(n-1)}{\cancel{2!}} = 6\cdot 2!$$

$$n(n-1) = 6\cdot 2\cdot 1$$

$$n(n-1) = 12$$

$$n^2 - n = \cancel{12}$$
$$-12 \quad \cancel{-12}$$

$$n^2 - 1n - 12 = 0$$

$-4 \times 3 = -12$

$-4 + 3 = -1$

$$(n-4)(n+3) = 0$$

$n - \cancel{4} = 0$ or $n + \cancel{3} = 0$

$+\cancel{4} \quad +4 \qquad -\cancel{3} \quad -3$

$\boxed{n = 4}$ or $\cancel{n = -3}$ reject