4.5 – Exploring Combinations

Date: Feb. 29

Combinations: A grouping of object where order does not matter.

Permutations are used to track when order matters.

BYRG
RBYG

Example: Four different coloured balls are placed in a bag. How many different arrangements can be made of the four coloured balls?

F.C.P.

$4 \times 3 \times 2 \times 1$ OR Permutation Function

$4! = 24$ $\quad {}_4P_4 = 24$

Combinations are used when order does not matter.

Example: A committee of four is being chosen from four students. How many different committees can be created?

Alf, Bob, Cam, Don → all get picked
order doesn't matter
only ONE possible committee*

*assumes people are not assigned to specific positions

Combinations and permutations use a special **notation** for their calculation.

${}_nC_r$ or $\binom{n}{r}$ "n choose r"

${}_nP_r$ "n permutes r"

We can use permutations to simplify a combination question.

Example: A committee of five is being chosen from ten candidates. How many unique committees can be chosen?

$${}_{10}P_5 = \frac{10!}{(10-5)!} = 30\,240$$

includes all different arrangements of each set of 5 people (eg. ABCDE, BAECD, BCDAE...)

$5! = 120$ ← how many ways we can arrange each set of 5 people

$$\text{unique committees} = \frac{30\,240}{120} = 252$$

We can use the combinations formula or the Fundamental Counting Principle to solve.

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

Example: A pizza shop has 12 different toppings. How many unique three-topping pizzas can the store offer?

$_{12}C_3 = \boxed{220}$ different 3 topping pizzas.

Same as: $\frac{_{12}P_3}{_3P_3} = 220$

Example: A coach from a soccer team of 15 players must choose 11 players to start the game. Of the 11 players, one must be chosen as field captain. How many ways can the coach make these selections?

not assigning specific positions → order doesn't matter.

Pick starters

$_{15}C_{11} = 1365$ ways to choose starters.

AND

Pick captain.

11 ways to choose captain

$1365 \times 11 = \boxed{15015}$ ways the coach can make the selections.