

4.4 – Permutations When Objects Are Identical

Date: Feb. 27

When objects are identical, certain arrangements repeat the exact same pattern.

Example: Three towels are being stacked in a closet. Two are identically red, and one is blue. What possible arrangements can be created?

$$\frac{3 \times 2 \times 1}{2 \times 1} = \boxed{3 \text{ arrangements}}$$

 ← 2 the same

$$\text{if unique } 3 \times 2 \times 1 = 6 \text{ arrangements}$$

Diagrams showing arrangements of towels (B for blue, R₁ and R₂ for red):

 1. B, R₁, R₂

 2. R₁, B, R₂

 3. R₂, R₁, B

Example: How many different ways can the letters in the word 'PAPPA' be arranged?

When there are n items in a group, and 'a' items are identical, 'b' items are identical, etc. then the total number of possible arrangements is given by:

$$P = \frac{n!}{a! b! c! d! \dots}$$

PAPPA $n = 5$
 $a = 3$ 'P'
 $b = 2$ 'A'

$$\frac{5!}{3! 2!} = \boxed{10} \text{ different arrangements}$$

 (5 letters, 3 Ps, 2 As)

Example: How many different arrangements can be made of the word 'SARSPARILLA'?

$n = 11$
 $a = 2$ 'S'
 $b = 3$ 'A'
 $c = 2$ 'R'
 $d = 2$ 'L'

$$\frac{11!}{2! 3! 2! 3!} = \boxed{831600} \text{ different arrangements.}$$

 (2 Ss, 3 As, 2 Rs, 2 Ls)

Example: A multiple choice test has 10 questions. How many different versions of the test can be created if four use 'a' as the answer, three use 'b' as the answer, two use 'c' as the answer, and one uses 'd' as the answer?

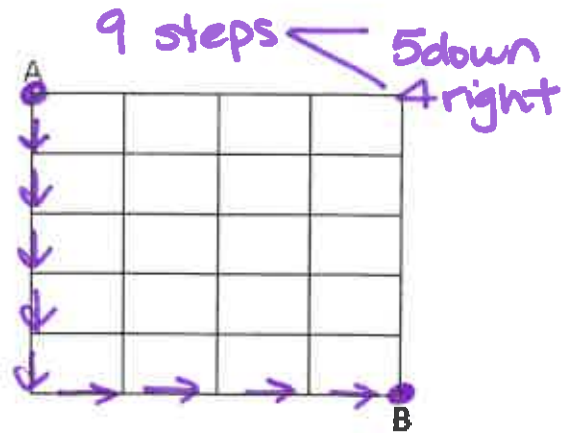
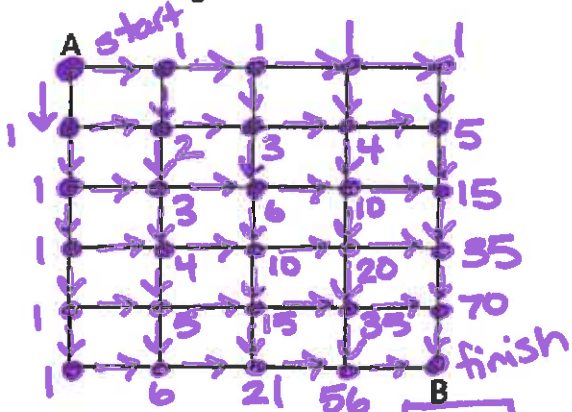
$n = 10$
 $a = 4$
 $b = 3$
 $c = 2$
 $d = 1$

$$\frac{10!}{4! 3! 2! 1!} = \boxed{12600} \text{ different versions.}$$

We can also use permutations to solve route problems.

DDDDDRRRR

Example: How many different ways can you move from A to B if you can only move down or right?



LOGIC

"count" the number of ways to get to each intersection

126

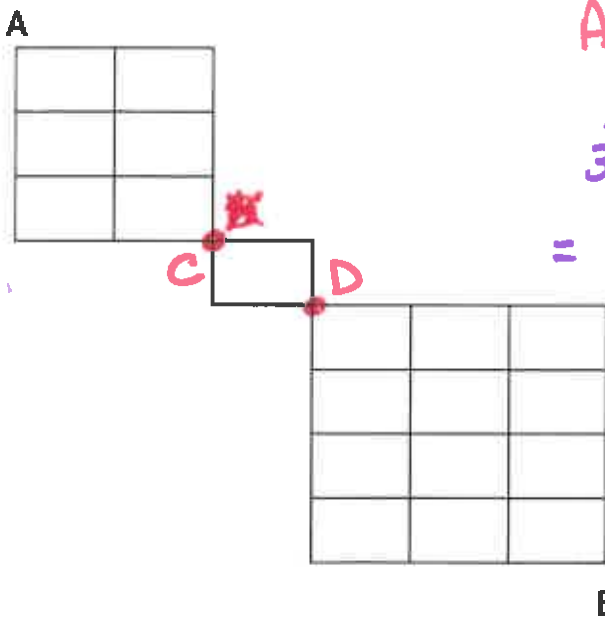
OR

PERMUTATIONS

$$\frac{9!}{5!4!}$$

= 126

Example: How many different ways can you move from A to B if you can only move down or right?



A → C and C → D and D → B

$$\frac{5!}{3!2!} \times \frac{2!}{1!1!} \times \frac{7!}{4!3!}$$

$$= 10 \times 2 \times 35$$

= 700 different routes.