4.4 – Permutations When Objects Are Identical

Date: Feb. 27

When objects are identical, certain arrangements repeat the exact same pattern.

Example: Three towels are being stacked in a closet. Two are identically red, and one is blue. What possible arrangements can be created?

B R R | R B R | R R B

B B | R_2 R_1 | R_2 R_1
R_2 R_1 | B B | R_1 R_2
R_1 R_2 | R_1 R_2 | B B

2 towels the same

$\frac{3\times2\times1}{2\times1} = 3$ arrangements

if unique (3 different towels) $3\times2\times1 = 6$ arrangements

Example: How many different ways can the letters in the word 'PAPPA' be arranged?

> When there are n items in a group, and 'a' items are identical, 'b' items are identical, etc., then the total number of possible arrangements is given by:
>
> c, d...
>
> $$P = \frac{n!}{a!\,b!\,c!\,d!\ldots}$$

PAPPA

$n = 5$ (5 letters)
$a = 3$ 'P'
$b = 2$ 'A'

$\frac{5!}{(3!\,2!)} = \boxed{10}$ different arrangements

3 Ps, 2 As

Example: How many different arrangements can be made of the word 'SARSPARILLA'?

$n = 11$
$a = 2$ 'S'
$b = 3$ 'A'
$c = 2$ 'R'
$d = 2$ 'L'

$\frac{11!}{(2!\,3!\,2!\,2!)} = \boxed{831\,600}$ different arrangements

Ss, As, Rs, Ls

Example: A multiple choice test has 10 questions. How many different versions of the test can be created if four use 'a' as the answer, three use 'b' as the answer, two use 'c' as the answer, and one uses 'd' as the answer?

$n = 10$
$a = 4$
$b = 3$
$c = 2$
$d = 1$

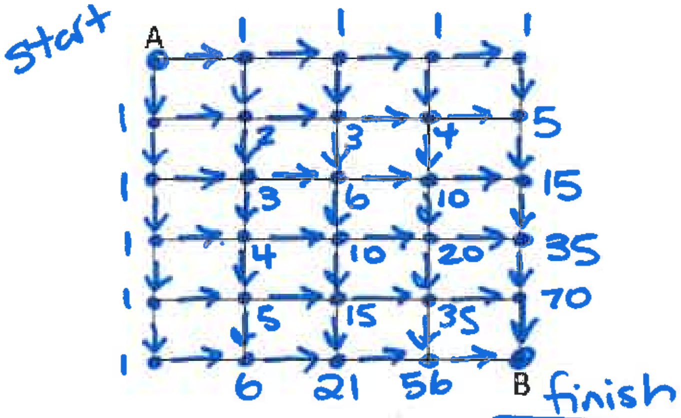
$\frac{10!}{(4!\,3!\,2!)} = \boxed{12\,600}$ different versions.

How many ways can we arrange these moves?

We can also use permutations to solve route problems.

DDDDDRRRR

Example: How many different ways can you move from A to B if you can only move down or right?



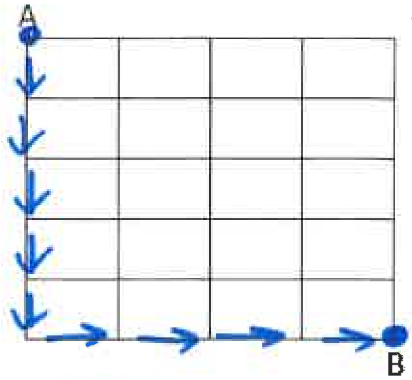
126 routes

LOGIC

"count" the number of ways to get to each intersection.

OR

9 moves ← 5 down, 4 across "right"

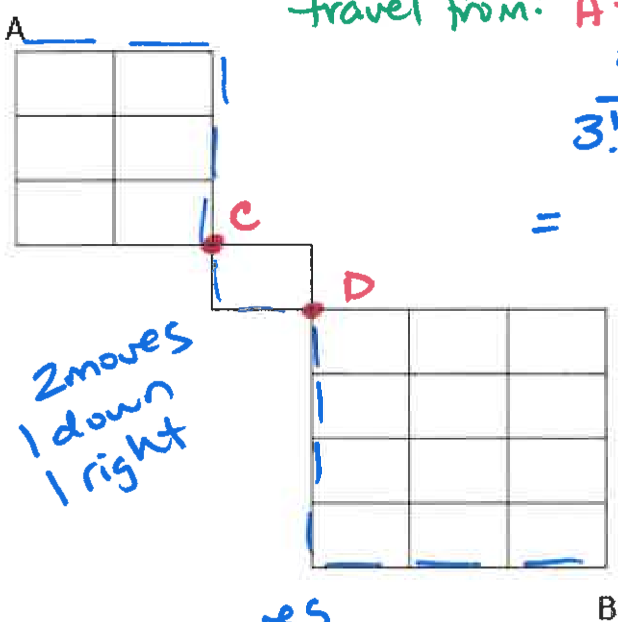


PERMUTATION

$$\frac{9!}{5!4!} = \boxed{126} \text{ routes}$$

Example: How many different ways can you move from A to B if you can only move down or right?

We need to travel from: A → C and C → D and D → B



5 moves, 3 down, 2 right

2 moves, 1 down, 1 right

7 moves, 4 down, 3 right.

$$\frac{5!}{3!2!} \times \frac{2!}{1!1!} \times \frac{7!}{4!3!}$$

$$= 10 \times 2 \times 35$$

$$= \boxed{700} \text{ different routes.}$$