4.3 – Permutations When All Objects Are Distinguishable

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Example: How many different four digit addresses can we make if we purchase the numbers 8, 7, 6, 5, 4, 3, and 2 at the hardware store?

a. Solve using the fundamental counting principal.

7 choices (only one of each number)

$7 \times 6 \times 5 \times 4 = \boxed{840}$

b. Solve using factorial notation.

$\frac{7!}{3!} = \boxed{840}$

c. Solve using permutation notation.

When we are selecting some objects out of a group *where each member of the group is distinct*, we can use the permutation function to calculate the number of possible results.

$${}_nP_r = \frac{n!}{(n-r)!}$$

n = number of different objects we are chosing from

r = number of objects we are selecting

$${}_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \boxed{840}$$

Example: How many different three letter combinations can be created from the word 'PRINCE'?

6 different letters

select 3

${}_6P_3 = \boxed{120}$

Example: 10 students are running for the positions of student council. Council includes one president, one vice-president, one secretary, and one treasurer. How many different student councils could be created?

10 students

4 positions

${}_{10}P_4 = \boxed{5040}$

Example: Solve ${}_7P_7 = \boxed{5040}$ $= \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{7!}{1} = 5040$

Example: When creating a password, the digits 0-9 can be used, as can any letters of the alphabet. Because the password is case sensitive, capital letters are different than lower case letters. How many unique passwords can be created that are either 4 or 5 characters if each character can only be used once?

+

no repetitions

\# of possible characters $= \underset{\text{digits}}{10} + \underset{\text{u.c. letter}}{26} + \underset{\text{l.c. letter}}{26} = 62$

4 character passwords : ${}_{62}P_4 = 13\ 388\ 280$

+

OR 5 character passwords : ${}_{62}P_5 = 776\ 520\ 240$

total: $\boxed{789\ 908\ 520}$

Example: A family of five is having portraits taken. How many ways can the photographer line up the family members assuming:

a. The two parents are together MD DM parents $\rightarrow {}_2P_2$

treat parents as one "block" in a four block group $\rightarrow {}_4P_4$

1 2 3 4

use FCP $\rightarrow {}_2P_2 \cdot {}_4P_4 = 2 \cdot 24 = \boxed{48}$

b. The parents must be on either end.

$\underset{\text{parent}}{\underline{2}} \times \underbrace{\underline{3} \times \underline{2} \times \underline{1}}_{\text{kids}} \times \underset{\text{parent}}{\underline{1}} = \boxed{12}$

OR

arrange parents ${}_2P_2 \cdot {}_3P_3$ arrange kids

$= 2 \cdot 6 = \boxed{12}$

c. The youngest child must be immediately between the parents

parents + youngest child. $\left.\begin{matrix} \underline{M}\ C\ \underline{D} \\ \underline{D}\ C\ \underline{M} \end{matrix}\right\}{}_2P_2$

2 3

treat as one "block" in a three block group. $\rightarrow {}_3P_3$

use FCP $\rightarrow {}_2P_2 \cdot {}_3P_3 = 2 \times 6 = \boxed{12}$

LNL NLN

26 letters

10 digits

Example: When creating postal codes, there are three letters and three numbers. How many postal codes can be created:

a. Allowing letters and numbers to be repeated

$$\underset{L}{26} \times \underset{N}{10} \times \underset{L}{26} \times \underset{N}{10} \times \underset{L}{26} \times \underset{N}{10} = \boxed{17\ 576\ 000}$$

b. Not allowing letters or numbers to be repeated

$$\underset{L}{26} \times \underset{N}{10} \times \underset{L}{25} \times \underset{N}{9} \times \underset{L}{24} \times \underset{N}{8}$$

OR

$${}_{26}P_3 \cdot {}_{10}P_3 = 15600 \times 720 = \boxed{11\ 232000}$$

c. How many more exist when repetition is allowed?

$$17\ 576\ 000 - 11\ 232\ 000 = \boxed{6\ 344\ 000}$$

Example: Solve for n, given $\boxed{{}_nP_2 = 90}$

$$\boxed{{}_nP_r = \frac{n!}{(n-r)!}}$$

$$\frac{n!}{(n-2)!} = 90$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 90$$

$$n(n-1) = 90$$

$$n^2 - n = 90$$
$$-90 \quad -90$$

$$n^2 - 1n - 90 = 0$$

$$-10 \times 9 = -90$$
$$-10 + 9 = -1$$

$$(n-10)(n+9) = 0$$

$$n - 10 = 0 \qquad n + 9 = 0$$
$$+10 \quad +10 \qquad -9 \quad -9$$

$$\boxed{n = 10} \qquad \cancel{n = -9}$$

reject.

$$\boxed{{}_{10}P_2 = 90}$$

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Example: Solve for r, given ${}_8P_r = 1680$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$\frac{\cancel{(8-r)!}}{1} \times \frac{8!}{\cancel{(8-r)!}} = 1680 \times (8-r)!$$

$$\frac{8!}{1680} = \frac{\cancel{1680}(8-r)!}{\cancel{1680}}$$

$1680 \div 8 = 210$
$210 \div 7 = 30$
$30 \div 6 = 5$

$8 \times 7 \times 6 \times 5 = 1680$

$$\frac{8!}{8 \times 7 \times 6 \times 5} = (8-r)!$$

$$\frac{\cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times 4!}{\cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5}} = (8-r)!$$

$$4! = (8-r)!$$

$$4 = 8 - r$$
$$-8 \quad -8$$

$$-4 = -r$$

$$\boxed{4 = r}$$

${}_8P_4 = 1680$