

4.2 – Introducing Permutations and Factorial Notation

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Permutation: An arrangement of distinguishable objects in a definite order.

For example, the objects a and b have two permutations: ab and ba .

Factorial notation: An abbreviated way of writing a series of multiplications of consecutive descending natural numbers using an exclamation point.

e.g. $5! = 5 \times 4 \times 3 \times 2 \times 1$

$N = \{1, 2, 3, 4, \dots\}$

Example: Record the total possible number of outcomes for three students in a lineup. What if we add a fourth? ABCD

Three people	Four people
ABC ACB BAC BCA CAB CBA	$ABCD$ $BACD$ $CABD$ $DABC$ $ABDC$ $BADC$ $CADB$ $DACB$ $ACBD$ $BCAD$ $CBAD$ $DBAC$ $ACDB$ $BCDA$ $CBDA$ $DBCA$ $ADBC$ $BDAC$ $CDAB$ $DCAB$ $ADCB$ $BDCA$ $CDAB$ $DCBA$
<div style="border: 1px solid black; padding: 5px; display: inline-block;">6 outcomes</div>	<p>4 groups of 6 =</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;">24 outcomes</div>

Example: Use the fundamental counting principal to solve the same lineup of 3 and 4 people.

three people: $\underline{3} \times \underline{2} \times \underline{1} = \underline{6 \text{ outcomes}}$ 3!

four people: $\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = \underline{24 \text{ outcomes}}$ 4!

Note that when we are using unique objects, the fundamental counting principal is always multiplying consecutively decreasing numbers.

We can use factorial notation to summarize this multiplication. !

Example: Write the following in factorial notation.

a. $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = \boxed{10!}$

b. $6 \times 5 \times 4 \times 3 \times 2 \times 1 = \boxed{6!}$

c. $8 \times 7 \times 6 \times 5 \times 4 = \frac{8 \times 7 \times 6 \times 5 \times 4 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1}} = \boxed{\frac{8!}{3!}}$

Why do we stop before zero?

anything $\times 0 = 0$ (0 is not a natural number).

Example: Evaluate the following.

a. $5! = \boxed{120}$

b. $\frac{9!}{6!} = \frac{9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}} = 9 \times 8 \times 7 = \boxed{504}$

c. $\frac{14!}{(10!2!)} = \frac{14 \times 13 \times 12 \times 11 \times \cancel{10!}}{\cancel{10!} \times 2!} = \frac{14 \times 13 \times 12 \times 11}{2 \times 1} = \boxed{12012}$

d. $n! = \boxed{n(n-1)(n-2) \dots (3)(2)(1)}$

e. $(-6)! = \text{does not exist.}$

f. $(0)! = \boxed{1}$

$$\begin{aligned} 2! &= \frac{3!}{3} \\ &= \frac{3 \times 2 \times 1}{3} \\ &= 2 \end{aligned}$$

$$\begin{aligned} 1! &= \frac{2!}{2} \\ &= \frac{2 \times 1}{2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} 0! &= \frac{1!}{1} \\ &= \frac{1}{1} \\ &= 1 \end{aligned}$$

Sometimes we want to solve a factorial expression algebraically. We can use what we know about factorials to simplify.

Example: Simplify the following:

$$7 \times 6! = 7!$$

a. $(x+4)(x+3)! = \frac{(x+4)(x+3)(x+2)(x+1)\dots}{(x+4)!}$

b. $\frac{(x+2)!}{(x-1)!} = \frac{(x+2)(x+1)(x)(x-1)!}{(x-1)!}$
 $= (x+2)(x+1)(x)$
 $= (x^2 + 1x + 2x + 2)(x)$
 $= (x^2 + 3x + 2)(x) = x^3 + 3x^2 + 2x$

Example: Solve the following:

a. $\frac{n!}{(n-2)!} = 42$

$$\frac{n!}{(n-2)!} = 42$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 42$$

$$n(n-1) = 42$$

$$n^2 - n = 42$$

$$\begin{aligned} -7 \times 6 &= -42 \\ -7 + 6 &= -1 \end{aligned}$$

$$n^2 - 1n - 42 = 0$$

$$(n-7)(n+6) = 0$$

$$n-7=0$$

OR

$$n+6=0$$

$$n=7$$

$$n=-6$$

reject!
NOT A
NATURAL
NUMBER

One last note! BE CAREFUL!

$$(n-2)! = (n-2)(n-3)!$$

~~NOT $(n-2)(n-1)!$~~