4.2 – Introducing Permutations and Factorial Notation

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Permutation: An arrangement of distinguishable objects in a definite order.

For example, the objects *a* and *b* have two permutations: *ab* and *ba*.

Factorial notation: An abbreviated way of writing a series of multiplications of consecutive descending natural numbers using an exclamation point.

$N = \{1, 2, 3, 4 \ldots\}$

e.g. 5! = 5 x 4 x 3 x 2 x 1

Example: Record the total possible number of outcomes for three students in a lineup. ABC
What if we add a fourth? ABCD

Three people	Four people
ABC ACB BAC BCA CAB CBA 6 outcomes	ABCD B___ C___ D___ ABDC B___ C___ D___ ACBD B___ C___ D___ ACDB B___ C___ D___ ADBC B___ C___ D___ ADCB B___ C___ D___ 4 groups of 6 = 24 outcomes.

Example: Use the fundamental counting principal to solve the same lineup of 3 and 4 people.

three people: $3 \times 2 \times 1 = \boxed{6 \text{ outcomes}}$ 3!

four people: $4 \times 3 \times 2 \times 1 = \boxed{24 \text{ outcomes}}$ 4!

Note that when we are using unique objects (can't repeat), the fundamental counting principal is always multiplying consecutive decreasing. numbers.

We can use factorial notation to summarize this multiplication.

Example: Write the following in factorial notation.

a. $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = \boxed{10!}$

b. $6 \times 5 \times 4 \times 3 \times 2 \times 1 = \boxed{6!}$

c. $\dfrac{8 \times 7 \times 6 \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}} = \boxed{\dfrac{8!}{3!}}$

Why do we stop before zero? (0 is not a natural number)

anything $\times 0 = 0$

Example: Evaluate the following.

a. $5! = \boxed{120}$

b. $\dfrac{9!}{6!} = \dfrac{9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}} = \boxed{504}$

c. $\dfrac{14!}{(10!2!)} = \dfrac{\overset{7}{\cancel{14}} \times 13 \times 12 \times 11 \times \cancel{10!}}{\cancel{10!} \times \cancel{2} \times 1} = \boxed{12012}$

d. $n! = \boxed{n(n-1)(n-2)\ldots(3)(2)(1)}$

e. $(-6)! =$ does not exist.

f. $(0)! = \boxed{1}$

Sometimes we want to solve a factorial expression algebraically. We can use what we know about factorials to simplify.

Example: Simplify the following:

a. $(x+4)(x+3)! = (x+4)(x+3)(x+2)(x+1)(x)\ldots$

$= \boxed{(x+4)!}$

b. $\frac{(x+2)!}{(x-1)!} = \frac{(x+2)(x+1)(x)\cancel{(x-1)!}}{\cancel{(x-1)!}}$

$= (x+2)(x+1)(x)$

$= (x^2 + 1x + 2x + 2)(x)$

$= (x^2 + 3x + 2)(x) = \boxed{x^3 + 3x^2 + 2x}$

Example: Solve the following:

a. $\frac{n!}{(n-2)!} = 42$

$\frac{n!}{(n-2)!} = 42$

$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 42$

$n(n-1) = 42$

$n^2 - n = 42$

$-42 \quad -42$

$n^2 - 1n - 42 = 0$

$(n-7)(n+6) = 0$

$-7 \times 6 = -42$

$-7 + 6 = -1$

$n - 7 = 0 \qquad n + 6 = 0$

$+7 \quad +7 \qquad -6 \quad -6$

$\boxed{n = 7}$ OR $\cancel{n = -6}$

reject

NOT A NATURAL NUMBER

One last note! BE CAREFUL!

(n – 2)! = $(n-2)(n-3)\ldots$

NOT $\cancel{(n-2)(n-1)}\ldots$