4.1 – Counting Principles

Date: Sept. 26

When we have multiple ways of accomplishing a task, we can use different ways to represent all the possible outcomes.

Example: The Spruce Kings typically have three colour jerseys: blue, white, and black. The tape on their stick can be white, black, or coloured. How many different ways can a Spruce King player dress for a hockey game?

Outcome table:

9 possible outfits

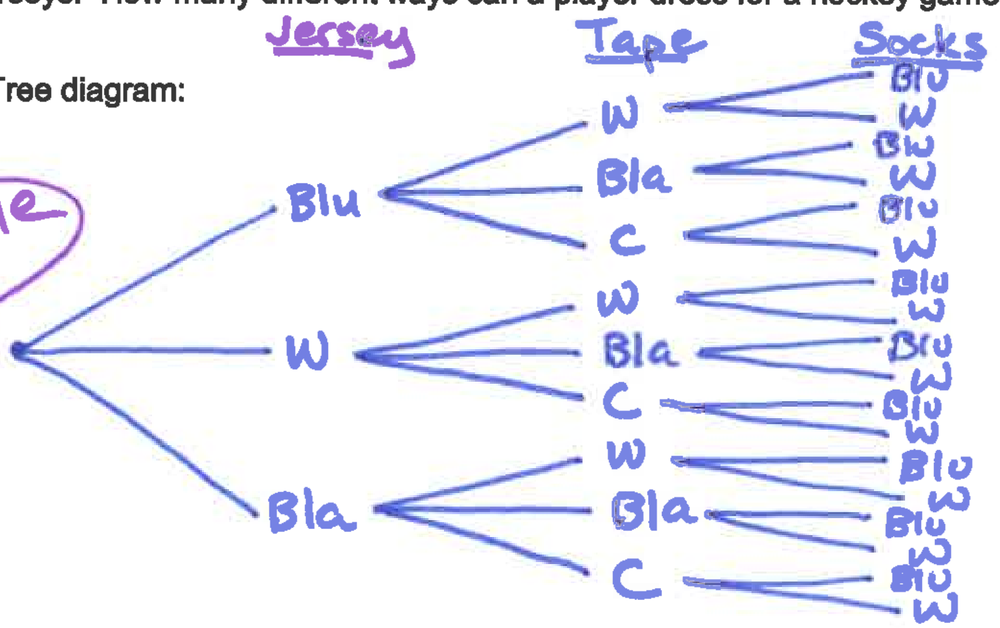
Tape

Jersey	White	Black	Colour
Blue	Blu/W	Blu/Bla	Blu/C
White	W/W	W/Bla	W/C
Black	Bla/W	Bla/Bla	Bla/C

Example: The players are also allowed to choose to wear either blue or white socks with their jerseys. How many different ways can a player dress for a hockey game now?

Tree diagram:

18 possible outfits



Example: Repeat both of the above calculations without using an outcome table or tree diagram.

$3 \times 3 = 9$ outfits

jersey options   tape options

$3 \times 3 \times 2 = 18$ outfits

jersey options   tape options   sock options

Fundamental Counting Principle:

If there are ___a___ ways to perform one task and ___b___ ways to perform another, then there are ___a · b___ ways of performing both.

Example:

A three digit combination lock has 40 possible numbers. How many possible different combinations can the padlock use?

Repetition allowed.

$40 \times 40 \times 40 = \boxed{64\,000}$

different combinations

A five digit combination lock uses the numbers 0-9 on each spool. Does this lock have more or less possible combinations than the previous lock?

Repetition allowed.

$10 \times 10 \times 10 \times 10 \times 10 = \boxed{100\,000}$

different combinations.

$\boxed{\text{More}}$ than previous lock.

Example: What if a digit cannot be repeated in either of the previous locks? Which has more possible combinations?

No repetition

$\underline{40} \times \underline{39} \times \underline{38} = \boxed{59\,280}$

possible combinations

No repetition

$10 \times 9 \times 8 \times 7 \times 6 = \boxed{30\,240}$

possible combinations

Example: A test has fifteen True/False questions. How many unique test answer sets are possible? T or F → 2 outcomes for each question

$\underset{Q1}{2} \times \underset{Q2}{2} \times \underset{Q3}{2} \ldots$

$2^{15} = \boxed{32\,768}$

possible answer sets.

Some situations cannot be solved using the Fundamental Counting Principal.

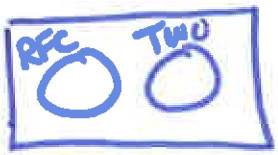
We cannot use the Fundamental Counting Principal when the word "or" is used to relate the outcomes.

Example: How many ways can you draw either a red face card or a two from a standard deck of playing cards?

RFC: J♥ Q♥ K♥ J♦ Q♦ K♦ → 6 outcomes

Two: 2♥ 2♦ 2♠ 2♣ → 4 outcomes

MUTUALLY EXCLUSIVE



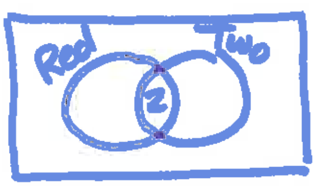
$6 + 4 = \boxed{10 \text{ outcomes}}$

Example: How many ways can you draw either a red card or a two?

Red Cards: 26 → 2 are Twos

Twos: 4 → 2 are red

overlap!



$$n(R \cup T) = n(R) + n(T) - n(R \cap T)$$
$$= 26 + 4 - 2$$
$$= \boxed{28 \text{ outcomes}}$$

Example: How many four digit numbers that are larger than 6600 can be formed from the digits 1, 3, 4, 6, 8 and 9 if repetition is not allowed?

Scenario 1 (8 or 9 as the first digit)

$\underset{8 \text{ or } 9}{\underline{2}} \times \underline{5} \times \underline{4} \times \underline{3} = 120$

OR

Scenario 2 (6 as the first digit)

$\underset{6}{\underline{1}} \times \underset{8 \text{ or } 9}{\underline{2}} \times \underline{4} \times \underline{3} = 24$

$120 + 24 = \boxed{144}$ possibilities