

A Venn diagram can be used to present conditional statements. The hypothesis will be a subset of the conclusion. → if the statement is true.

Example: If ^{hyp.} (pull a heart from a deck of cards), ^{concl.} it will be a red card.

U = {set of all cards in a deck}

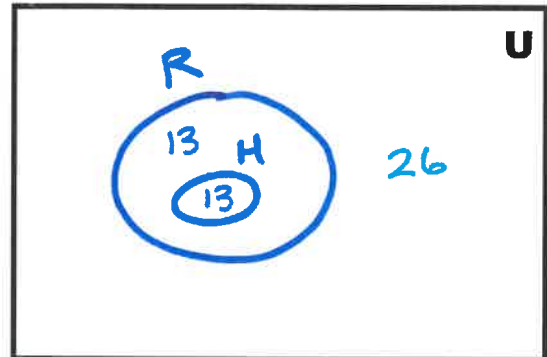
R = {set of all red cards}

H = {set of all hearts}

If H, then R.

$H \subset R$

TRUE!



3.6 – The Inverse and the Contrapositive of Conditional Statements

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Inverse: A statement that is formed by negating both the hypothesis and the conclusion of a conditional statement.

Contrapositive: A statement that is formed by negating both the hypothesis and the conclusion of the converse of a conditional statement.

Example: State the hypothesis and conclusion, then write the converse, inverse and contrapositive.

If you eat an apple each day, you keep the doctor away.

Hypothesis: you eat an apple each day

Conclusion: you keep the doctor away.

Converse: If you keep the doctor away,
you eat an apple each day.

Inverse: If you do not eat an apple each day,
you do not keep the doctor away.

Contrapositive: If you do not keep the doctor away,
you do not eat an apple each day.

Switch

Negate

Switch and Negate

		Reverse Order
	Conditional	Converse
Negate	Inverse	Contrapositive

Statement	Behaviour
Conditional	If p, then q.
Converse	If q, then p.
Inverse	If not p, then not q.
Contrapositive	If not q, then not p.

Key Ideas

- If a conditional statement is true, then its contrapositive is also true.
- If the inverse of a statement is true, then the converse of the statement is also true.

See example below!

We can use these relationships to help prove or disprove a statement.

Example: Write the converse, inverse and contrapositive. Evaluate if each is true or false. If possible, write as a biconditional statement.

If a colour is red, yellow or blue, it is a primary colour. TRUE

Converse: If a colour is a primary colour, it is red, yellow, or blue. TRUE

Inverse: If a colour is not red, yellow or blue, it is not a primary colour.

Contrapositive: If a colour is not a primary colour, it is not red, yellow or blue.

Biconditional: A colour is red, yellow or blue if and only if it is a primary colour.

ORIGINAL STATEMENT:

If x^2 is even, then x is even. cannot prove deductively.

CONTRAPOSITIVE:

If x is not even, then x^2 is not even.

x is odd. x^2 is odd.

We can prove that our original statement is true by proving that the contrapositive is true.

let $2n+1 = \text{odd number}$ ← x is odd

$$\begin{aligned} (2n+1)^2 &= (2n+1)(2n+1) \\ &= 4n^2 + 2n + 2n + 1 \\ &= 4n^2 + 4n + 1 \\ &= 2(2n^2 + 2n) + 1 \end{aligned}$$

← x^2 is odd.