3.5 – Conditional Statements and Their Converse

hypothesis

Date: Feb. 12/13

If it is Monday then it is a school day. FALSE

conclusion

Conditional statement: An "if-then" statement

Hypothesis: An assumption used within a conditional statement.

Conclusion: The result of a hypothesis.

Counterexample: An example that disproves a statement. eg. No school next Monday (Family day)

Converse: A conditional statement in which the hypothesis and the conclusion are switched.

If it is a school day, then it is Monday. FALSE

Biconditional: A true conditional statement whose converse is also true.

> If-then statements create a conditional statement with the conditional hypothesis determining the truth of the conclusion. These statements can be read as:
>
> **If p, then q.**
>
> **p implies q.**
>
> **p → q.** $p \Rightarrow q$

Example: Identify the hypothesis and conclusion of the following statement:

If a number is even, then it is divisible by two

Hypothesis: a number is even TRUE

Conclusion: it is divisible by two

Example: Identify the hypothesis and conclusion of the following statement:

y=8 if y-3=5

Hypothesis: $y-3=5$ TRUE

Conclusion: $y=8$

> A conditional statement is false only when the hypothesis is true and the conclusion.
>
> see example p.195
>
> Otherwise the conditional statement is true, even if the hypothesis is false. When a conditional statement is false, we can present a counterexample which disproves the statement.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: Identify the hypothesis and conclusion of the following statement. Determine if it is true or false. If false, provide a counterexample.

You must be a doctor if you work in a hospital.

Hypothesis: you work in a hospital

Conclusion: you must be a doctor

True $\boxed{\text{False}}$

Counterexample: I could be a nurse and work in a hospital.

The converse of a conditional statement switches the hypothesis with the conclusion. For a conditional statement of "If p, then q.", the converse statement would read:

If q, then p.

q implies p.

q → p.

* Note that the truth of a converse statement is independent of the truth of a conditional statement. *

Example: Identify the hypothesis and conclusion of the following statement. Determine if it is true or false. If false, provide a counterexample. Then write the converse statement, and determine if it is true or false.

If a room is painted red, the room is painted with a warm colour.

Hypothesis: a room is painted red.

Conclusion: the room is painted with a warm colour.

$\boxed{\text{True}}$ False

Counterexample: none exist

Converse: If the room is painted with a warm colour, then ~~a~~ the room is painted red.

True $\boxed{\text{False}}$

Counterexample: A room painted yellow is a warm colour.

Example: Identify the hypothesis and conclusion of the following statement. Determine if it is true or false. If false, provide a counterexample. Then write the converse statement, and determine if it is true or false.

If $x^2 \geq 0$ *then* $x \geq 0$.

Hypothesis: $x^2 \geq 0$

Conclusion: $x \geq 0$

True $\boxed{\text{False}}$

Counterexample: $x = -1$ $x^2 = 1$
not ≥ 0

Converse: If $x \geq 0$, then $x^2 \geq 0$.

$\boxed{\text{True}}$ False

Counterexample: none exist.

A biconditional statement occurs when both the conditional statement and its converse statement are true. When this occurs, we can write the biconditional statement as:

p if and only if q.

p ↔ q.

Example: Identify the hypothesis and conclusion of the following statement. Write the converse. Determine if the statement is biconditional, and write as a biconditional if it exists.

If a right triangle contains a 30° angle, the right triangle also contains a 60° angle.

Hypothesis: a right triangle contains a 30° angle

Conclusion: the right △ also contains a 60° angle.

$\boxed{\text{True}}$ False

Converse: If a right triangle contains a 60° angle, then the right triangle also contains a 30° angle.

$\boxed{\text{True}}$ False

Biconditional: A right triangle contains a 30° angle if and only if the right triangle also contains a 60° angle.

A Venn diagram can be used to present conditional statements. The hypothesis will be a subset of the conclusion. → if the statement is true.

hyp. conc.

Example: If I pull a heart from a deck of cards, it will be a red card.

U = {set of all cards in a deck}

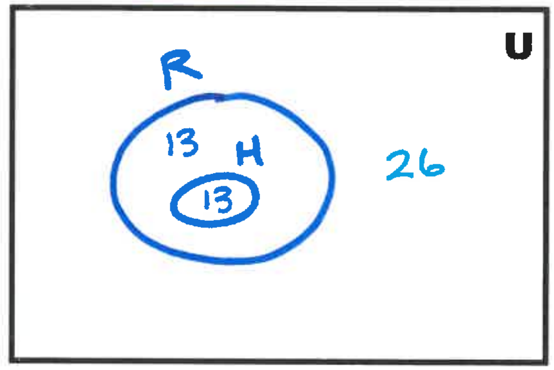
R = {set of all red cards}

H = {set of all hearts}

If H, then R.

$H \subset R$

TRUE!



3.6 – The Inverse and the Contrapositive of Conditional Statements

Date: ____________________

Inverse: A statement that is formed by negating both the hypothesis and the conclusion of a conditional statement.

Contrapositive: A statement that is formed by negating both the hypothesis and the conclusion of the converse of a conditional statement.

Example: State the hypothesis and conclusion, then write the converse, inverse and contrapositive.

If you eat an apple each day, you keep the doctor away.

Hypothesis: ____

Conclusion: ____

Converse: ____

____

Inverse: ____

____

Contrapositive: ____

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