

3.4 – Applications of Set Theory

Date: Feb. 8

Example: Rachel surveyed Grade 12 students about how they communicated with friends over the previous week.

% $n(U) = 100$

- $n(C) =$ • 66% called on a cellphone **C**
- $n(T) =$ • 76% texted **T**
- $n(S) =$ • 34% used a social networking site **S**
- $n(C \cap T) =$ • 56% called on a cellphone and texted
- $n(C \cap S) =$ • 18% called on a cellphone and used a social networking site
- $n(T \cap S) =$ • 19% texted and used a social networking site, and
- $n(C \cap T \cap S) =$ • 12% used all three forms of communication.

→ find $n(C \cup T \cup S)$

What percentage of students used at least one of these three forms of communication? Represent using a Venn Diagram.

start in the middle! $n(C \cap T \cap S) = 12$

$$n(C \cap T \setminus S) = 56 - 12 = 44$$

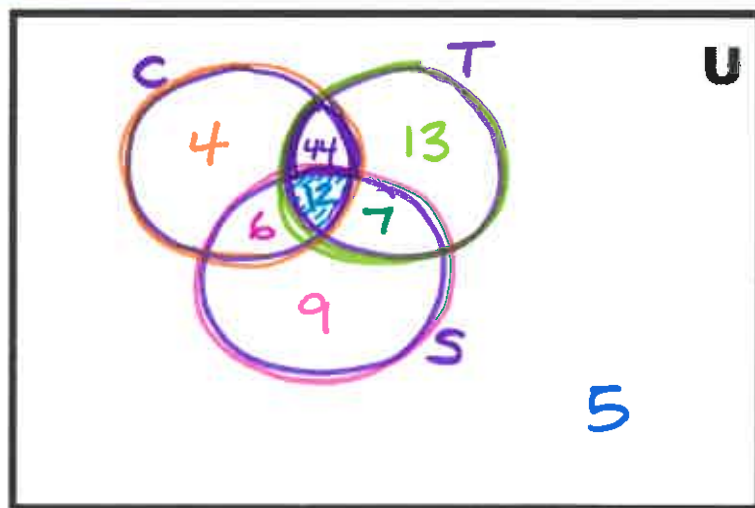
$$n(T \cap S \setminus C) = 19 - 12 = 7$$

$$n(C \cap S \setminus T) = 18 - 12 = 6$$

$$n(C \setminus T \setminus S) = 66 - 44 - 12 - 6 = 4$$

$$n(T \setminus C \setminus S) = 76 - 44 - 12 - 7 = 13$$

$$n(S \setminus T \setminus C) = 34 - 7 - 12 - 6 = 9$$



$$n(C \cup T \cup S) = 4 + 13 + 9 + 44 + 7 + 6 + 12 = 95$$

95% of students use at least one form of communication

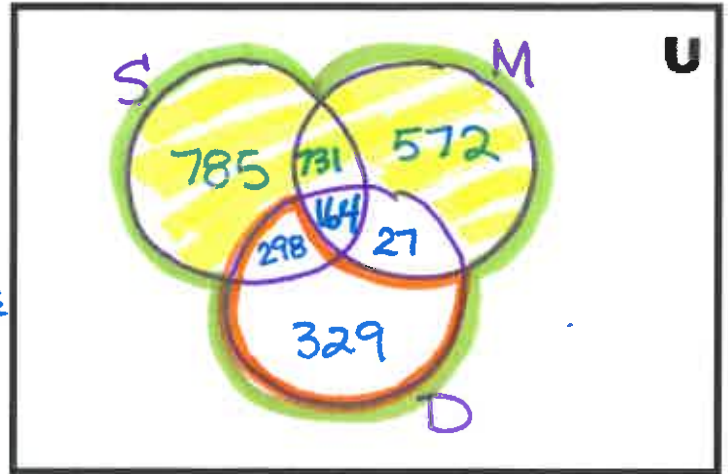
$100 - 95 = 5$
5% didn't use technology.

FORMULA: $n(C \cup T \cup S) = n(C) + n(T) + n(S) - n(C \cap T) - n(C \cap S) - n(T \cap S) + n(C \cap T \cap S)$

$$n(U) = 3000$$

Example: A board game company completed a survey of 3000 customers to find out which board games they owned. The top three were as follows: Scrabble (S), Monopoly (M), and Dominos (D).

- $n(S)$ • 1978 owned Scrabble
- $n(M)$ • 1494 owned Monopoly
- $n(D)$ • 818 owned Dominos
- $n(S \cap M \setminus D)$ • 731 owned both Scrabble and Monopoly only *
- $n(S \cap D \setminus M)$ • 298 owned Scrabble and Dominos only *
- $n(M \cap D \setminus S)$ • 27 owned Monopoly and Dominos only *
- $n(S \cap M \cap D)$ • 164 people owned all three games. *



* can plug right into the diagram

- a. Create a Venn diagram of the survey results.

$$n(S \setminus M \setminus D) = 1978 - 731 - 298 - 164 = 785$$

$$n(M \setminus S \setminus D) = 1494 - 731 - 27 - 164 = 572$$

$$n(D \setminus M \setminus S) = 818 - 298 - 27 - 164 = 329$$

- b. How many people owned Dominos but not Monopoly?

$$= 329 + 298 = \boxed{627}$$

- c. Find $n((S \cup M) \cup (M \cup D)) = n(S \cup M \cup D)$

$$= 785 + 572 + 329 + 731 + 27 + 298 + 164 = \boxed{2906}$$

people own at least one of the games

- d. How many people do not own any of these board games?

$$n(S \cup M \cup D)' = n(U) - n(S \cup M \cup D) = 3000 - 2906 = \boxed{94}$$

do not own any of the games

- e. Find $n((S \cup M) \setminus D)$

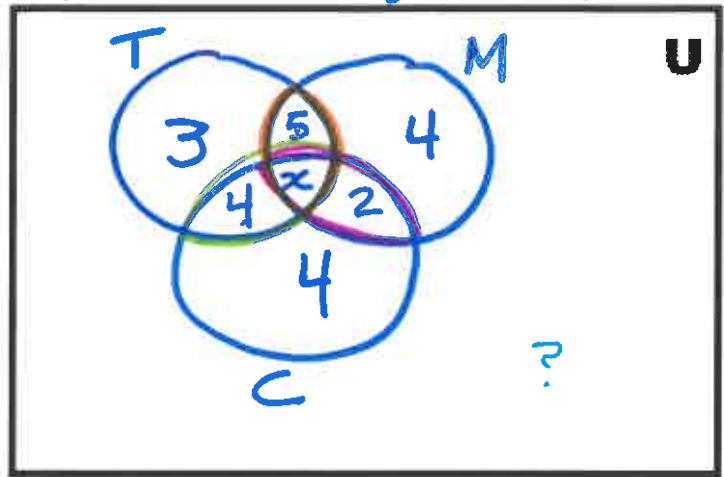
number of people who have S or M but not D

$$= 785 + 731 + 572 = \boxed{2088}$$

unknown
 $n(U)$ = number of students in class

Example: In a Grade 12 math class:

- $n(T \cup M \cup C)$ • 25 students have a tablet, an MP3 player, or a cellphone
- $n(T)$ • 15 students have a tablet
- $n(M)$ • 14 students have an MP3 player
- $n(C)$ • 13 students have a cellphone
- $n(T \cap M \setminus C)$ • 5 students have a tablet and an MP3 player, but not a cellphone
- $n(T \cap C \setminus M)$ • 4 students have a tablet and a cellphone, but not an MP3 player
- $n(M \cap C \setminus T)$ • 2 students have an MP3 player and a cellphone, but not a tablet



Represent this information using a Venn Diagram.

How many students have all three devices? = $n(T \cap M \cap C) = x$

How many students have only one device?

$$n(T \cup M \cup C) = n(T) + n(M) + n(C) - n(T \cap M) - n(M \cap C) - n(T \cap C) + n(T \cap M \cap C)$$

$$25 = 15 + 14 + 13 - (x+5) - (x+2) - (x+4) + x$$

$$25 = 42 - x - 5 - x - 2 - x - 4 + x$$

$$25 = 31 - 2x$$

$$-6 = -2x$$

$$3 = x$$

3 students have all three devices

$$n(T \setminus M \setminus C) = 15 - 5 - 3 - 4 = 3$$

$$n(M \setminus C \setminus T) = 14 - 5 - 3 - 2 = 4$$

$$n(C \setminus T \setminus M) = 13 - 4 - 3 - 2 = 4$$

$$3 + 4 + 4 = 11$$

11 students have only one device