3.3 – Intersection and Union of Two Sets

Date: Feb. 7

Intersection: The set of elements that are common to two or more sets.

Uses the word "AND".

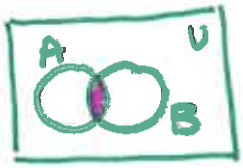
Union: The set of all the elements in two or more sets.

Uses the word "OR".

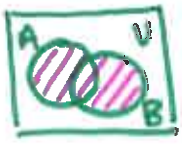
Principle of Inclusion or Exclusion: The number of elements in the union of two sets is equal to the sum of the number of elements in each set, less the number of elements in both sets.

Relationship	Elements Included	Set Notation	Set Equation for Solving
A intersect B	Elements in both A and B only	$A \cap B$	
A union B	Elements in either A or B	$A \cup B$	$n(A \cup B)=n(A)+n(B)-n(A \cap B)$ or $n(A \cup B)=n(A \setminus B)+n(B \setminus A)+n(A \cap B)$
A not B	Elements in A but not in B	$A \setminus B$	$n(A \setminus B)=n(A)-n(A \cap B)$
Not in A	All the elements not in A	A'	$n(A')=n(U)-n(A)$

$A \cap B$ ↑ intersection



$A \cup B$ ↑ union



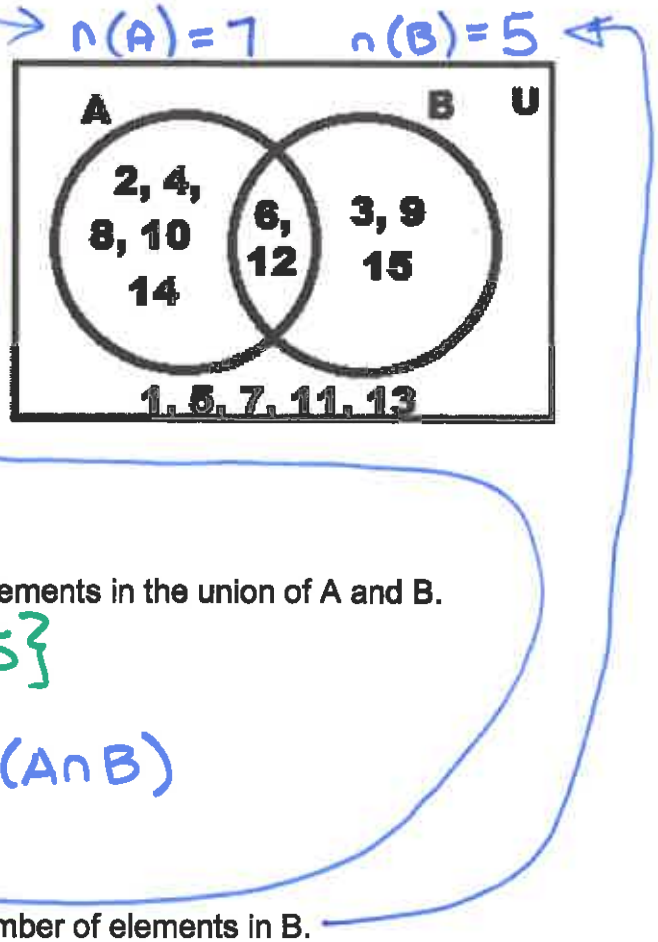
$A \setminus B$ ↑ not "minus"

Example:

Let U = {all numbers between and including 1 and 15}

Let A = {all multiples of two between 1 and 15}

Let B = {all multiples of three between 1 and 15}



Use set notation in your answers for the following questions.

a. Find the intersection of A and B.

~~A ∪ B~~ $A \cap B = \{6, 12\}$

$n(A \cap B) = 2$

b. Find the union of A and B and the number of elements in the union of A and B.

$A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15\}$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 7 + 5 - 2$

$= 10$

c. Find the number of elements in A. Find the number of elements in B.

d. Find the number of elements in A only. Find the number of elements in B only.

$n(A \setminus B) = n(A) - n(A \cap B)$

$= 7 - 2$

$= 5$

$n(B \setminus A) = n(B) - n(A \cap B)$

$= 5 - 2$

$= 3$

Example: Jamal surveyed 34 people at their gym. They learned that 16 people do weight training regularly, 21 people do cardio training regularly and 6 people do not do either. Create a Venn diagram, showing all calculations in set notation.

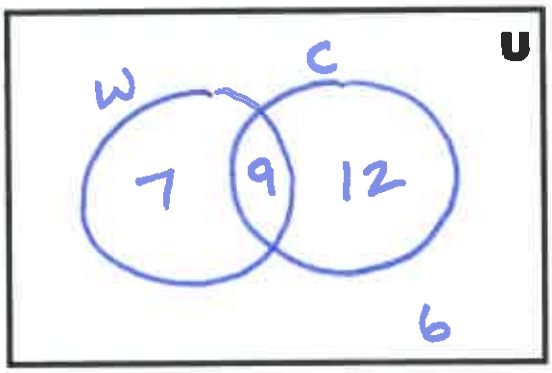
$n(U) = 34$

$n(W) = 16$

$n(C) = 21$

$34 - 6 = 28$ do one or both

$n(W \cup C) = 28$



$n(W \cup C) = n(W) + n(C) - n(W \cap C)$

$28 = 16 + 21 - n(W \cap C)$

$28 = 37 - n(W \cap C)$

$-37 \quad -37$

$-9 = -n(W \cap C)$

$9 = n(W \cap C)$

$n(W \setminus C) = n(W) - n(W \cap C)$

$= 16 - 9$

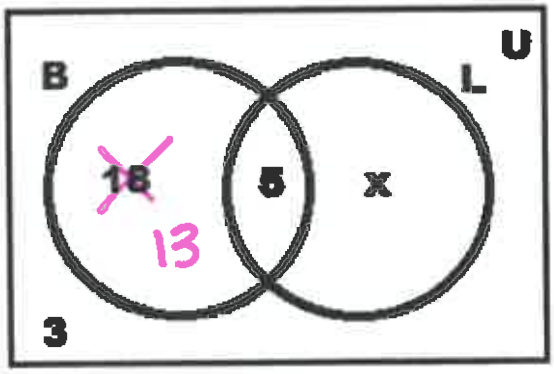
$= 7$

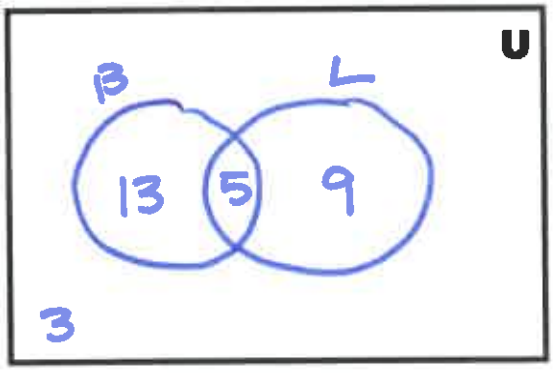
$n(C \setminus W) = n(C) - n(W \cap C)$

$= 21 - 9$

$= 12$

Example: Morgan surveyed 30 students in her school about their eating habits. 18 students eat breakfast. 5 of the 18 also eat a healthy lunch. 3 students do not eat either. How many students eat a healthy lunch? Identify the error in the Venn Diagram below and correct it.





$n(B \cup L) = 30 - 3$

$= 27$

$n(B \cup L) = n(B) + n(L) - n(B \cap L)$

$27 = 18 + n(L) - 5$

$27 = 13 + n(L)$

$\boxed{14} = n(L)$

$n(L \setminus B) = n(L) - n(L \cap B)$

$= 14 - 5$

$= 9$

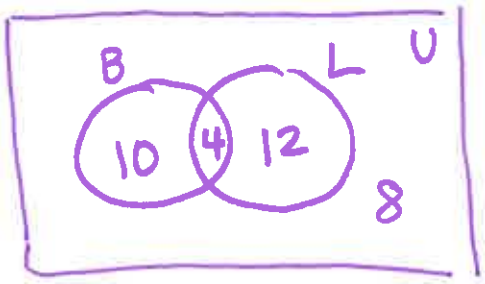
Example: Susan surveyed 34 students and found that 14 eat breakfast, 16 eat a healthy lunch, and 4 (of the above 30) eat both. She concluded that everyone eats breakfast, lunch, or both. Identify the error in her logic and find how many students do not eat either meal.

$n(U) = 34$

$n(B) = 14$

$n(L) = 16$

$n(B \cap L) = 4$.



* She did not account for the overlap! Can't just add 14 + 16 + 4 because the 4 people who eat both lunch & breakfast are counted twice!

$$n(B \setminus L) = n(B) - n(B \cap L)$$
$$= 14 - 4$$
$$= 10$$

$$n(L \setminus B) = n(L) - n(B \cap L)$$
$$= 16 - 4$$
$$= 12$$

$$\Rightarrow n(B \cup L) = n(B) + n(L) - n(B \cap L)$$
$$= 16 + 14 - 4$$
$$= 26$$

$$n(B \cup L)' = n(U) - n(B \cup L)$$
$$= 34 - 26$$
$$= 8$$

8 students do not eat either meal.